中国石油大学
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Madagascar experience

Application of lowrank decomposition for seismic wave modeling

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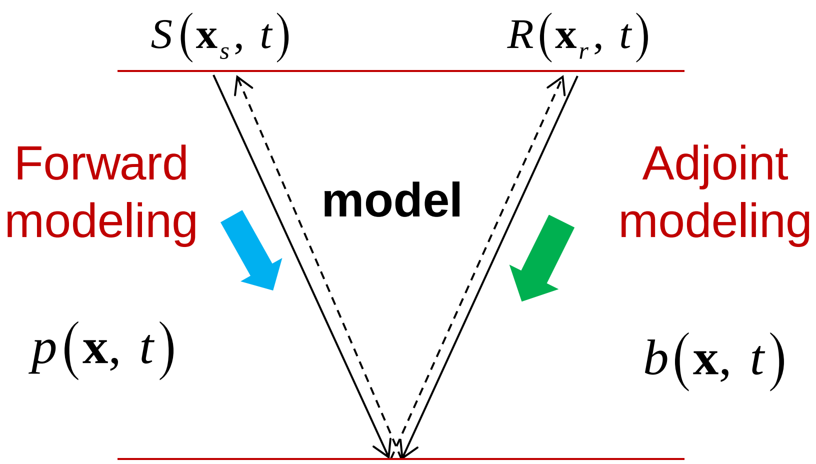
The role of seismic modeling in RTM and FWI

Reverse time migration (RTM)

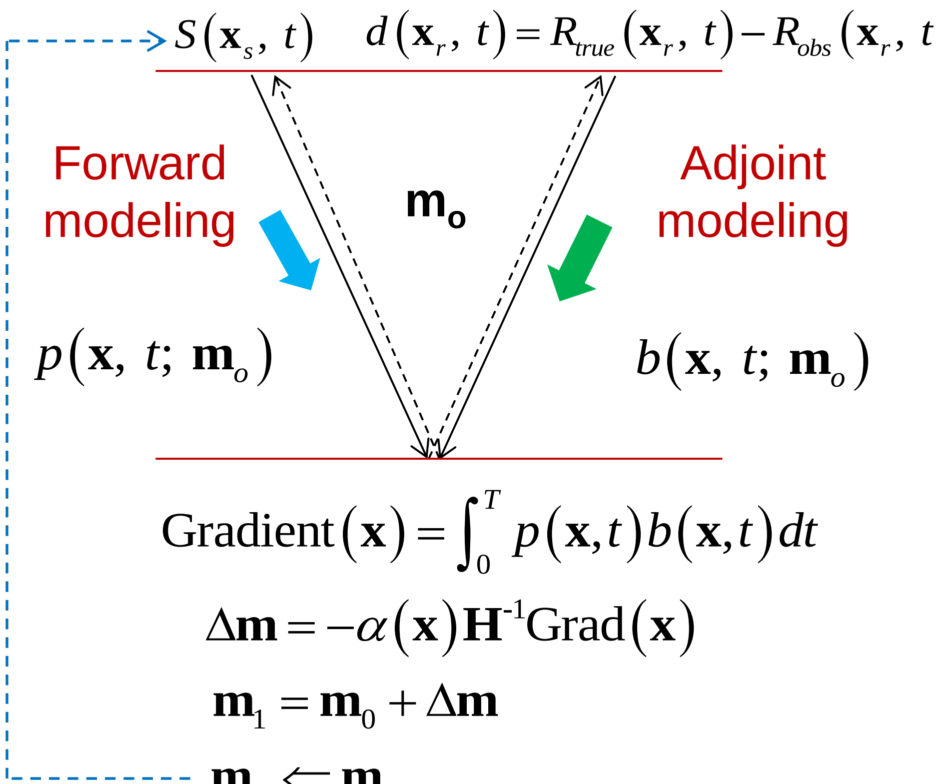


$$\text{Image}(\mathbf{x}) = \int_0^T p(\mathbf{x},t)\, b(\mathbf{x},t)\, dt$$

or other imaging conditions

- Essential core
- Critical for efficiency

Full waveform inversion (FWI)



$$\text{Gradient}(\mathbf{x}) = \int_0^T p(\mathbf{x},t)\, b(\mathbf{x},t)\, dt$$

$$\Delta\mathbf{m} = -\alpha(\mathbf{x})\,\mathbf{H}^{-1}\text{Grad}(\mathbf{x})$$

$$\mathbf{m}_1 = \mathbf{m}_0 + \Delta\mathbf{m}$$

$$\mathbf{m}_0 \Leftarrow \mathbf{m}_1$$

until $\left\| R_{true}(\mathbf{x}_r, t) - R_o(\mathbf{x}_r, t) \right\|_2 < \varepsilon$

Existing modeling code in Madagascar

- Lowrank and lowrank finite-difference methods
- Main published papers by TCCS

1. Fomel, S., L. Ying, and X. Song, 2013, Seismic wave extrapolation using lowrank symbol approximation: Geophysical Prospecting, **61**, 526–536.

2. Song, X., S. Fomel, and L. Ying, 2013, Lowrank finite-differences and lowrank Fourier finite-differences for seismic wave extrapolation: Geophysical Journal International, **193**, 960–969.

3. Fang, G., S. Fomel, Q. Du, and J. Hu, 2014, Lowrank seismic-wave extrapolation on a staggered grid: Geophysics, **79**, no. 3, T157–T168.

4. Cheng J. and S. Fomel, 2014, Fast algorithms for elastic-wave-mode separation and vector decomposition using low-rank approximation for anisotropic media: Geophysics, **79**, no. 4, C97–C110.

Acoustic lowrank extrapolation

- Exact time marching scheme

① $$\frac{\partial^2 p(\mathbf{x},\, t)}{\partial t^2} = v^2 \nabla^2 p(\mathbf{x},\, t)$$ ➡ ② $$\frac{\partial^2 \tilde{p}(\mathbf{k},t)}{\partial t^2} = v^2 |\mathbf{k}|^2 \tilde{p}(k,t)$$

③ $$\tilde{p}(\mathbf{k},t+\Delta t) = e^{\pm i|k|v\Delta t}\tilde{p}(\mathbf{k},t)$$ Analytical solution

④ $$p(\mathbf{x},t+\Delta t) + p(\mathbf{x},t-\Delta t) = 2IFFT\left\{\cos\left(v\Delta t|\mathbf{k}|\right)FFT\left[p(\mathbf{x},t)\right]\right\}$$

Constant velocity: one time FFT + one time IFFT

⑤ $$p(\mathbf{x},t+\Delta t) + p(\mathbf{x},t-\Delta t) = 2IFFT\left\{\cos\left(v(\mathbf{x})\Delta t|\mathbf{k}|\right)FFT\left[p(\mathbf{x},t)\right]\right\}$$

Variable velocity: one time FFT + N times IFFT (N denotes the number of distinct velocities)

Acoustic lowrank extrapolation

- Lowrank decomposition (S. Fomel et al., 2013, GP)

$$\mathbf{W}(\mathbf{x},\mathbf{k})_{N\times N} = \cos\left[|\mathbf{k}|v(\mathbf{x})\Delta t\right] \approx \mathbf{W}_1(\mathbf{x},\mathbf{k}_m)_{N\times m} \times \mathbf{A}_{m\times n} \times \mathbf{W}_2(\mathbf{x}_n,\mathbf{k})_{n\times N}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} \mathbf{W}_1(\mathbf{x},\mathbf{k}_m) a_{ij} \mathbf{W}_2(\mathbf{x}_n,\mathbf{k})$$

- Lowrank time marching scheme

$$p(\mathbf{x},t+\Delta t) + p(\mathbf{x},t-\Delta t) = 2IFFT\left\{\cos\left[|\mathbf{k}|v(\mathbf{x})\Delta t\right]FFT\left[p(\mathbf{x},t)\right]\right\}$$

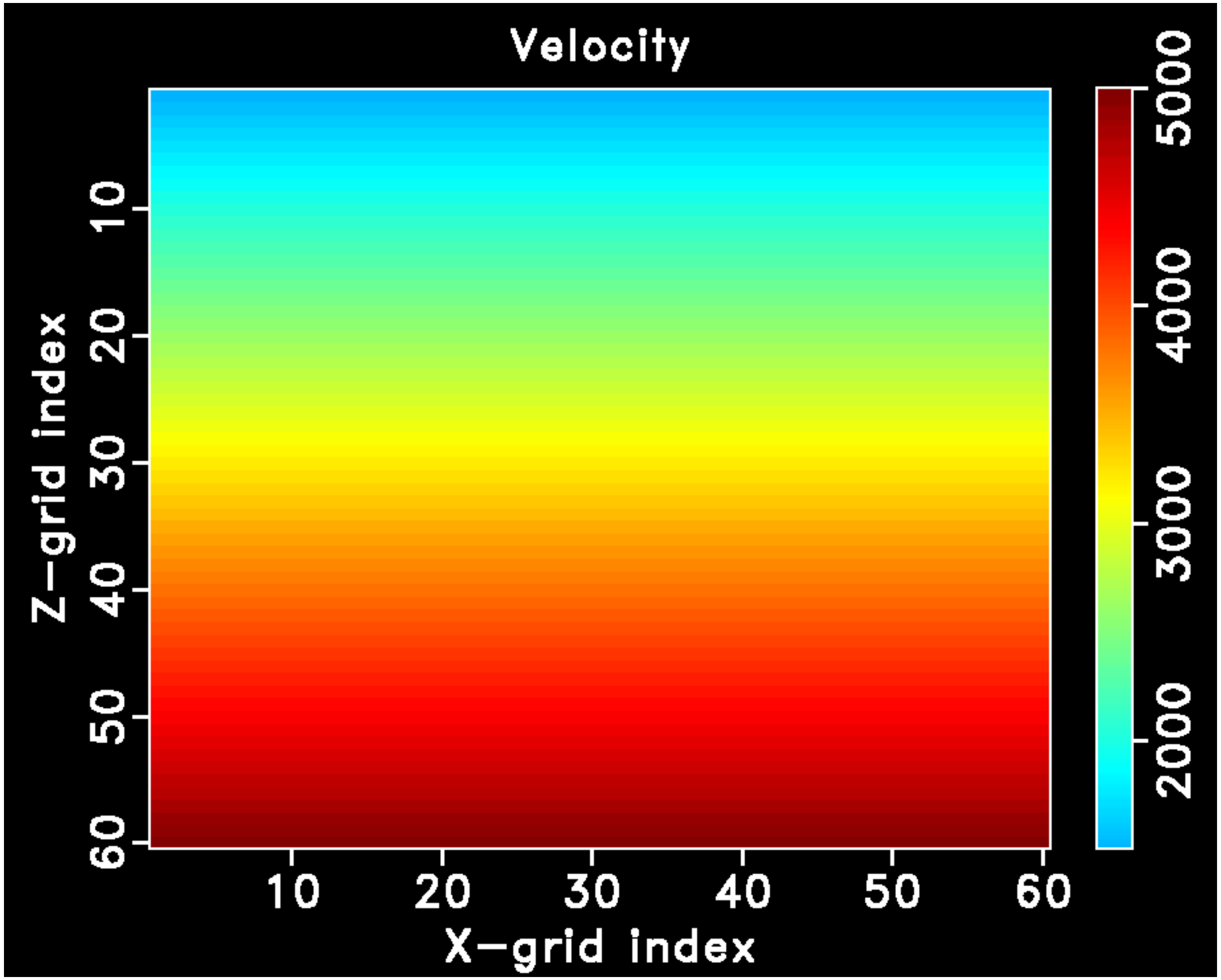
$$= 2\sum_{i=1}^{m} \mathbf{W}_1(\mathbf{x},\mathbf{k}_m) \sum_{j=1}^{n} a_{ij} IFFT\left[\mathbf{W}_2(\mathbf{x}_n,\mathbf{k}) FFT\left[p(\mathbf{x},t)\right]\right]$$

Variable velocity: one time FFT + *n* times IFFT

n is a small number

Acoustic lowrank extrapolation

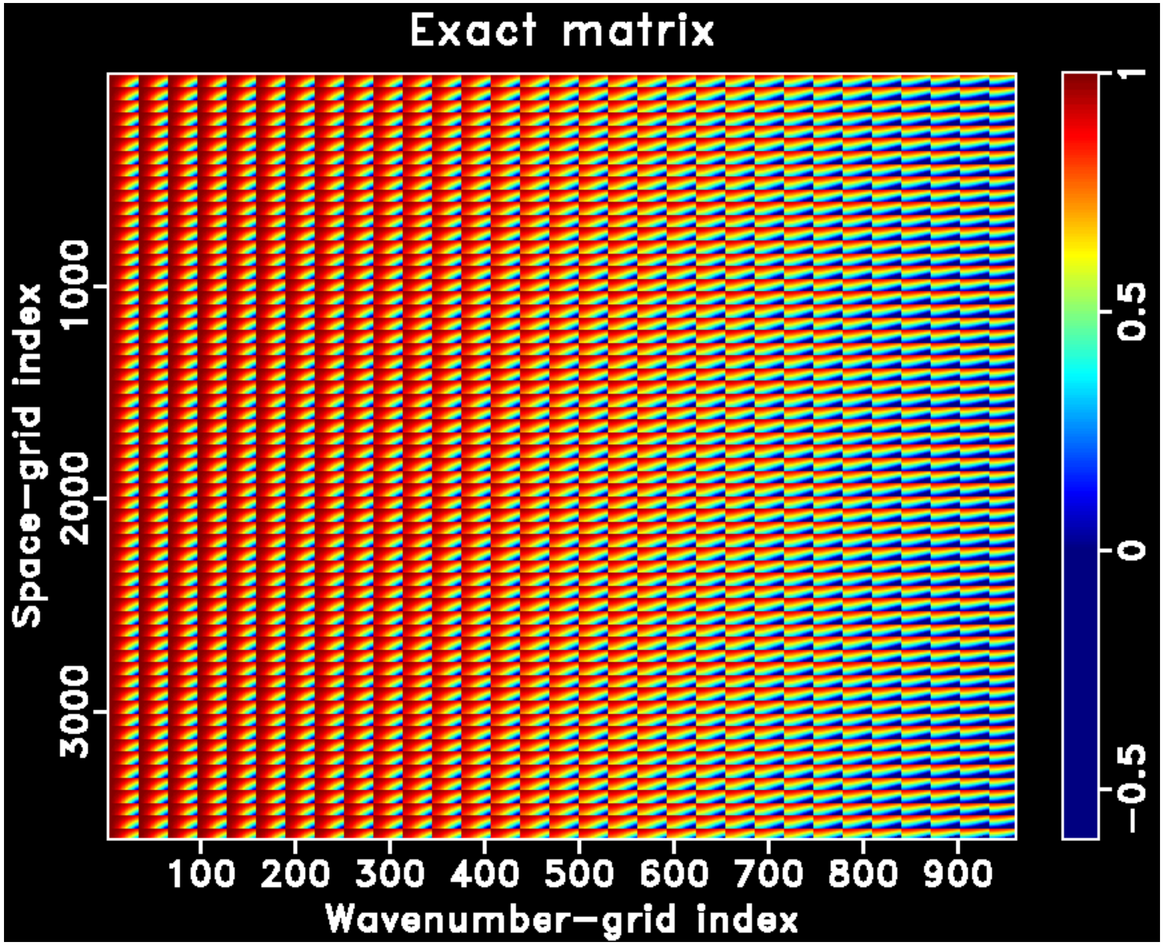
- Lowrank decomposition test (by my Madagascar C program)



$\Delta t = 1 \text{ ms}, h_x = h_z = 10 \text{ m}, v_{\min} = 1500 \text{ m/s}, v_{\max} = 5000 \text{ m/s}$

Acoustic lowrank extrapolation

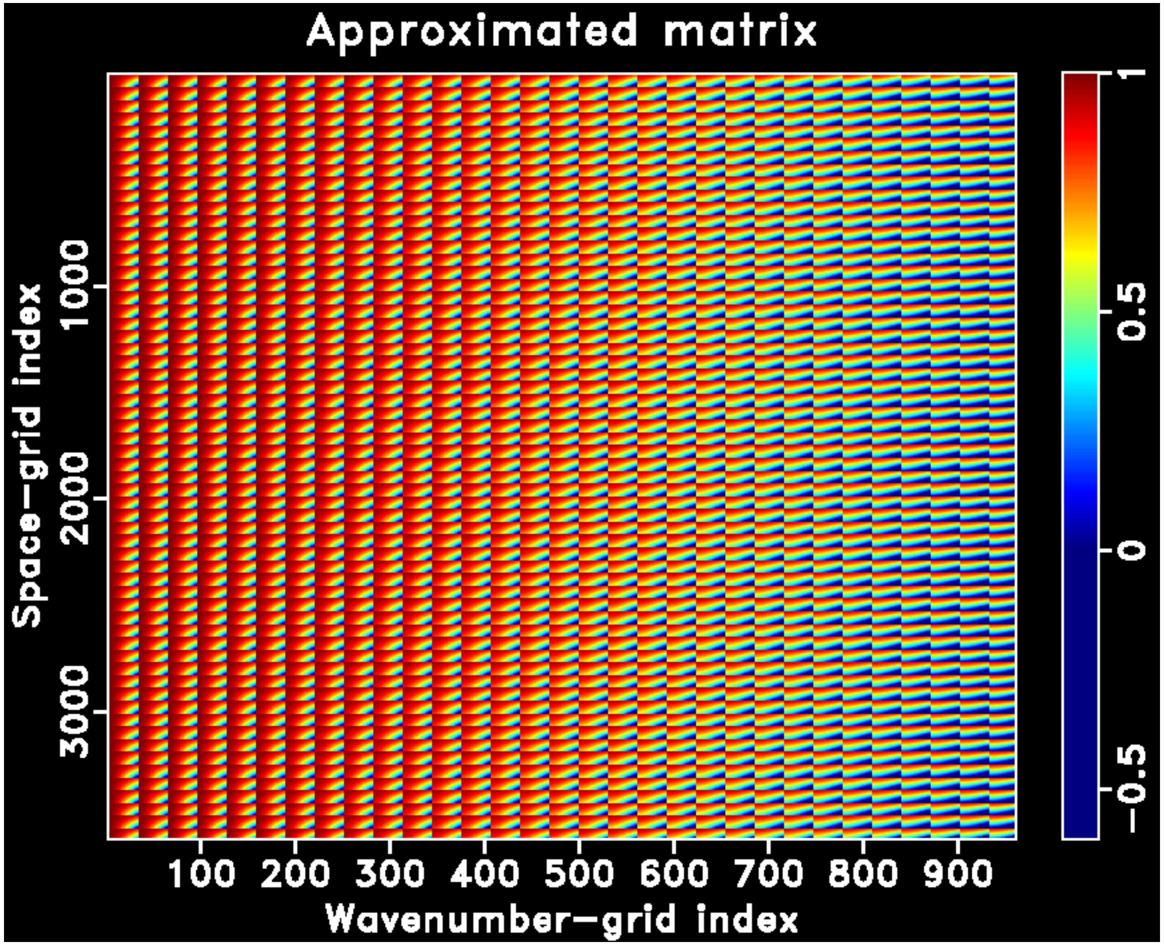
- Lowrank decomposition test (by my Madagascar C program)



$$\mathbf{W}(\mathbf{x},\mathbf{k})_{N\times N} = \cos\left[|\mathbf{k}|\, v(\mathbf{x})\,\Delta t\right]$$

Acoustic lowrank extrapolation

- Lowrank decomposition test (by my Madagascar C program)

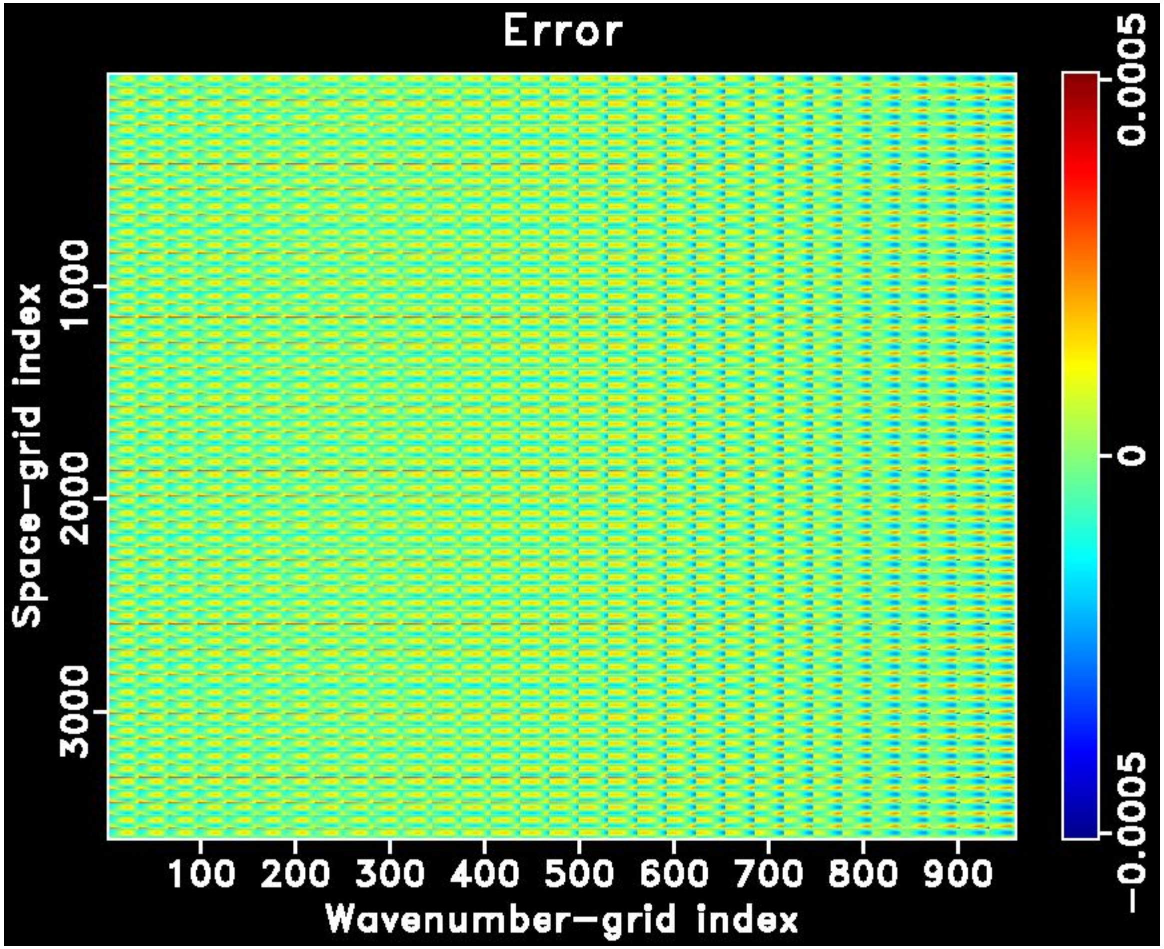


$m = n = 3$

$$\mathbf{W}(\mathbf{x},\mathbf{k})_{N\times N} = \mathbf{W}_1\mathbf{A}\mathbf{W}_2$$

Acoustic lowrank extrapolation

- Lowrank decomposition test (by my Madagascar C program)



$m = n = 3$

Small magnitude

Acoustic lowrank extrapolation

- Staggered-grid lowrank method (G. Fang et al., 2014)

$$\tilde{p}(\mathbf{k},t+\Delta t)+\tilde{p}(\mathbf{k},t-\Delta t)-2\tilde{p}(\mathbf{k},t)=2\left[\cos\left(v\Delta t\left|\mathbf{k}\right|\right)-1\right]\tilde{p}(\mathbf{k},t) \quad ①$$

⬇

$$\frac{\tilde{p}(\mathbf{k},t+\Delta t)-2\tilde{p}(\mathbf{k},t)+\tilde{p}(\mathbf{k},t-\Delta t)}{v^2\Delta t^2}=-\left(k_x^2+k_z^2\right)\mathrm{sinc}^2\left(\frac{v\left|\mathbf{k}\right|\Delta t}{2}\right)\tilde{p}(\mathbf{k},t) \quad ②$$

⬆

modified spectral response for 1st-order derivatives

$$\nabla^2=\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-}+\frac{\partial}{\partial z^+}\frac{\partial}{\partial z^-}$$

$$\begin{cases}\dfrac{\partial}{\partial x^+}=IFFT\left[ik_x\mathrm{sinc}\left(\dfrac{v\left|\mathbf{k}\right|\Delta t}{2}\right)e^{ik_xh_x/2}\right]\\ \dfrac{\partial}{\partial x^-}=IFFT\left[ik_x\mathrm{sinc}\left(\dfrac{v\left|\mathbf{k}\right|\Delta t}{2}\right)e^{-ik_xh_x/2}\right]\\ \dfrac{\partial}{\partial z^+}=IFFT\left[ik_z\mathrm{sinc}\left(\dfrac{v\left|\mathbf{k}\right|\Delta t}{2}\right)e^{ik_zh_z/2}\right]\\ \dfrac{\partial}{\partial z^-}=IFFT\left[ik_z\mathrm{sinc}\left(\dfrac{v\left|\mathbf{k}\right|\Delta t}{2}\right)e^{-ik_zh_z/2}\right]\end{cases}$$

applied to

$$\begin{cases}\rho\dfrac{\partial v_x}{\partial t}=-\dfrac{\partial p}{\partial x^+}\\ \rho\dfrac{\partial v_z}{\partial t}=-\dfrac{\partial p}{\partial z^+}\\ \dfrac{\partial p}{\partial t}=-K\left(\dfrac{\partial v_x}{\partial x^-}+\dfrac{\partial v_z}{\partial z^-}\right)\end{cases}$$

Acoustic lowrank extrapolation

- Lowrank FD method (X. Song et al., 2013)

$$\mathbf{W}(\mathbf{x},\mathbf{k})_{N\times N} = \cos\left[|\mathbf{k}|v(\mathbf{x})\Delta t\right] \approx \mathbf{W}_1(\mathbf{x},\mathbf{k}_m)_{N\times m} \times \mathbf{A}_{m\times n} \times \mathbf{W}_2(\mathbf{x}_n,\mathbf{k})_{n\times N} \quad ①$$

$$\mathbf{W}_2(\mathbf{x}_n,\mathbf{k})_{n\times N} = \mathbf{C}_{n\times L}\mathbf{B}_{L\times N} \quad ②$$

L denotes the FD stencil length, **B** represents the stencil matrix.

③ $\mathbf{C} = \left[\left(\mathbf{B}\mathbf{\Lambda}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{\Lambda}\mathbf{W}_2{}^T\right]^T$ weighted least-squares approach

Coefficient matrix for all spatial locations $\mathbf{x}$

$$\mathbf{G}_{N\times L} = \mathbf{W}_1 \times \mathbf{A} \times \mathbf{C} \quad ④$$

* FD stencil (critical for the accuracy of solving LS problem)

$$\nabla^2 p_{0,0} = \frac{1}{h^2}\left[c_0 p_{0,0} + \sum_{m=1}^{M} c_{m,0}\left(p_{m,0} + p_{-m,0} + p_{0,m} + p_{0,-m}\right) + c_{1,1}\left(p_{1,1} + p_{-1,1} + p_{1,1} + p_{1,-1}\right)\right]$$

off-axial grid points

$L = M + 2$

Elastic lowrank extrapolation

- Constant velocity and density isotropic elastic formulation

① $$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2} + \beta^2 \dfrac{\partial^2 u}{\partial z^2} + \left(\alpha^2 - \beta^2\right) \dfrac{\partial^2 w}{\partial xz} \\ \dfrac{\partial^2 w}{\partial t^2} = \beta^2 \dfrac{\partial^2 w}{\partial x^2} + \alpha^2 \dfrac{\partial^2 w}{\partial z^2} + \left(\alpha^2 - \beta^2\right) \dfrac{\partial^2 u}{\partial xz} \end{cases}$$

α P-wave velocity

β S-wave velocity

② $$\frac{\partial^2 \tilde{\mathbf{P}}(\mathbf{k},\ t)}{\partial t^2} = -\mathbf{L}\tilde{\mathbf{P}}(\mathbf{k},\ t),$$

③ $$\mathbf{L} = \begin{bmatrix} \alpha^2 k_x^2 + \beta^2 k_z^2 & \left(\alpha^2 - \beta^2\right) k_x k_z \\ \left(\alpha^2 - \beta^2\right) k_x k_z & \beta^2 k_x^2 + \alpha^2 k_z^2 \end{bmatrix}, \qquad \tilde{\mathbf{P}}(\mathbf{k},\ t) = \left[\tilde{u}(\mathbf{k},\ t), \tilde{w}(\mathbf{k},\ t)\right]^T$$

Elastic lowrank extrapolation

- Exact time marching scheme,

$$\tilde{\mathbf{P}}(t+\Delta t)+\tilde{\mathbf{P}}(t-\Delta t)=2\mathbf{D}\cdot\mathbf{P}(t) \qquad ①$$

Based on EVD,

$$\mathbf{D}=\cos\left(\mathbf{L}^{\frac{1}{2}}\Delta t\right)=\left[\mathbf{v}_1,\ \mathbf{v}_2\right]\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\left[\mathbf{v}_1,\ \mathbf{v}_2\right]^{-1} \qquad ②$$

$$\begin{cases}\lambda_1=\cos\left(\alpha\Delta t\left|\mathbf{k}\right|\right),\ \lambda_2=\cos\left(\beta\Delta t\left|\mathbf{k}\right|\right),\\ \mathbf{v}_1=\left(k_x,\ k_z\right)^T,\ \mathbf{v}_2=\left(-k_z,\ k_x\right)^T,\end{cases} \qquad ③$$

In the same approach as that for developing acoustic staggered-grid lowrank method, we can obtain

Elastic lowrank extrapolation

- Our quasi-stress-velocity formulation

①
$$\begin{cases} \rho \dfrac{\partial v_x}{\partial t} = \underbrace{\left(\dfrac{\partial \tau_{111}}{\partial x}\right)_{L_p} + \left(\dfrac{\partial \tau_{11}}{\partial x}\right)_{L_s}}_{\tau_{xx}} + \left(\dfrac{\partial \tau_{12}}{\partial z}\right)_{L_s} \\ \rho \dfrac{\partial v_z}{\partial t} = \left(\dfrac{\partial \tau_{12}}{\partial x}\right)_{L_s} + \underbrace{\left(\dfrac{\partial \tau_{111}}{\partial z}\right)_{L_p} + \left(\dfrac{\partial \tau_{22}}{\partial z}\right)_{L_s}}_{\tau_{zz}} \\ \dfrac{\partial \tau_{111}}{\partial t} = (\lambda + 2\mu)\left(\dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_z}{\partial z}\right)_{L_p} \\ \dfrac{\partial \tau_{11}}{\partial t} = -2\mu\left(\dfrac{\partial v_z}{\partial z}\right)_{L_s} \\ \dfrac{\partial \tau_{12}}{\partial t} = \mu\left(\dfrac{\partial v_z}{\partial x} + \dfrac{\partial v_x}{\partial z}\right)_{L_s} \\ \dfrac{\partial \tau_{22}}{\partial t} = -2\mu\left(\dfrac{\partial v_x}{\partial x}\right)_{L_s} \end{cases}$$

$\beta = 0$

$$\begin{cases} \rho \dfrac{\partial v_x}{\partial t} = \left(\dfrac{\partial \tau_{111}}{\partial x}\right)_{L_p} \\ \rho \dfrac{\partial v_z}{\partial t} = \left(\dfrac{\partial \tau_{111}}{\partial z}\right)_{L_p} \\ \dfrac{\partial \tau_{111}}{\partial t} = (\lambda + 2\mu)\left(\dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_z}{\partial z}\right)_{L_p} \end{cases}$$

acoustic wave equation

- Traditional formulation (Virieux, 1986)

②
$$\begin{cases} \rho \dfrac{\partial v_x}{\partial t} = \dfrac{\partial \tau_{xx}}{\partial x} + \dfrac{\partial \tau_{xz}}{\partial z} \\ \rho \dfrac{\partial v_z}{\partial t} = \dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \tau_{zz}}{\partial z} \\ \dfrac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu)\dfrac{\partial v_x}{\partial x} + \lambda \dfrac{\partial v_z}{\partial z} \\ \dfrac{\partial \tau_{zz}}{\partial t} = \lambda \dfrac{\partial v_x}{\partial x} + (\lambda + 2\mu)\dfrac{\partial v_z}{\partial z} \\ \dfrac{\partial \tau_{xz}}{\partial t} = \mu\left(\dfrac{\partial v_z}{\partial x} + \dfrac{\partial v_x}{\partial z}\right) \end{cases}$$

Elastic lowrank extrapolation

- Split quasi-stress-velocity formulation

$$\begin{cases} \rho \dfrac{\partial v_x^{\mathrm{P}}}{\partial t} = \left(\dfrac{\partial \tau_{111}}{\partial x}\right)_{L_p} \\ \rho \dfrac{\partial v_x^{\mathrm{S}}}{\partial t} = \left(\dfrac{\partial \tau_{11}}{\partial x}\right)_{L_s} + \left(\dfrac{\partial \tau_{12}}{\partial z}\right)_{L_s} \\ v_x = v_x^{\mathrm{P}} + v_x^{\mathrm{S}} \end{cases}$$

$$\begin{cases} \rho \dfrac{\partial v_z^{\mathrm{P}}}{\partial t} = \left(\dfrac{\partial \tau_{111}}{\partial z}\right)_{L_p} \\ \rho \dfrac{\partial v_z^{\mathrm{S}}}{\partial t} = \left(\dfrac{\partial \tau_{12}}{\partial x}\right)_{L_s} + \left(\dfrac{\partial \tau_{22}}{\partial z}\right)_{L_s} \\ v_z = v_z^{\mathrm{P}} + v_z^{\mathrm{S}} \end{cases}$$

③

$$\begin{cases} \dfrac{\partial \tau_{111}}{\partial t} = (\lambda + 2\mu)\left(\dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_z}{\partial z}\right)_{L_p} \\ \dfrac{\partial \tau_{11}}{\partial t} = -2\mu\left(\dfrac{\partial v_z}{\partial z}\right)_{L_s} \\ \dfrac{\partial \tau_{12}}{\partial t} = \mu\left(\dfrac{\partial v_z}{\partial x} + \dfrac{\partial v_x}{\partial z}\right)_{L_s} \\ \dfrac{\partial \tau_{22}}{\partial t} = -2\mu\left(\dfrac{\partial v_x}{\partial x}\right)_{L_s} \end{cases}$$

$\left(v_x^{\mathrm{P}},\ v_z^{\mathrm{P}}\right)$ only contain P-wave, while $\left(v_x^{\mathrm{S}},\ v_z^{\mathrm{S}}\right)$ only contain S-wave.

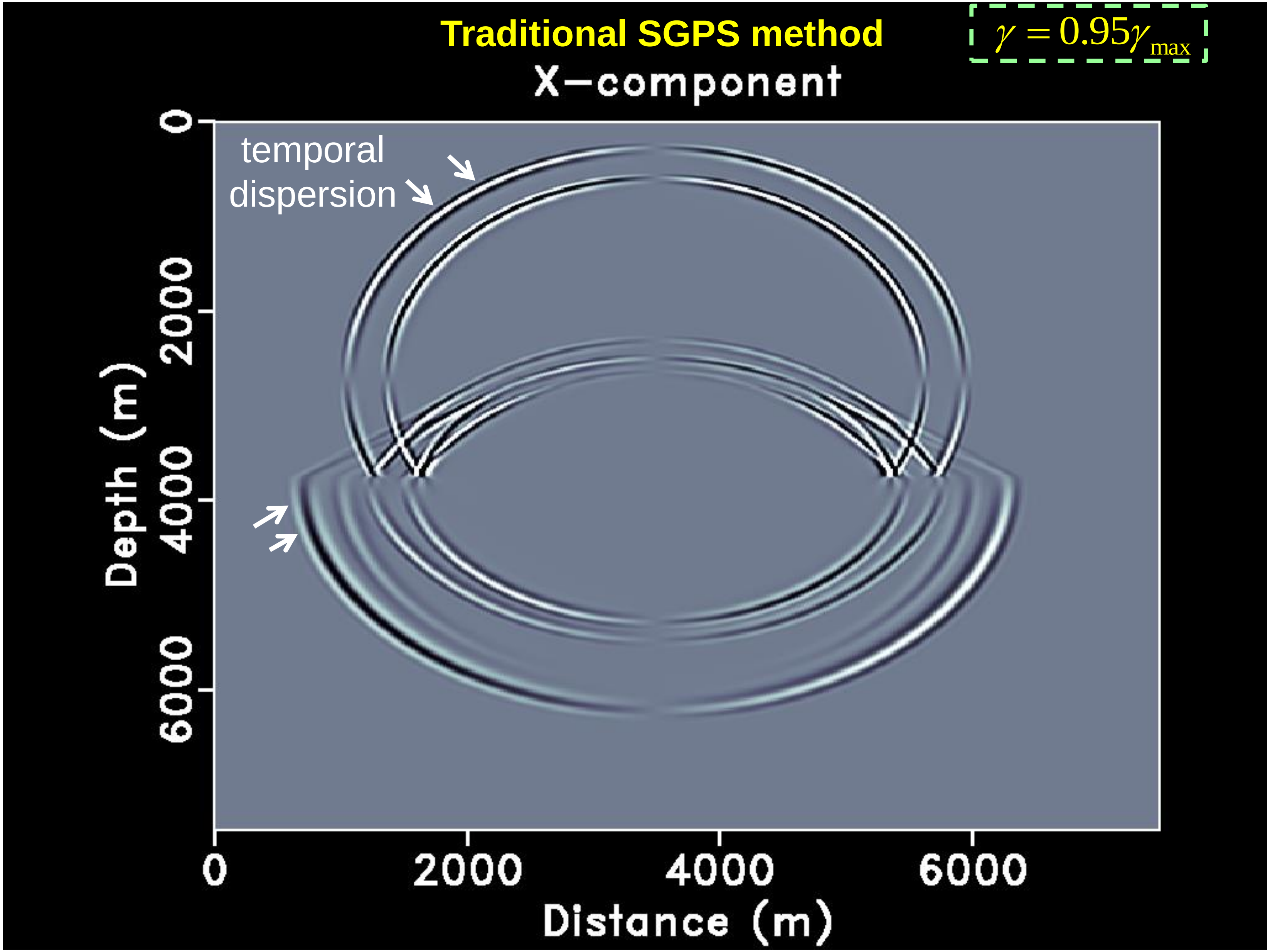
Elastic lowrank extrapolation

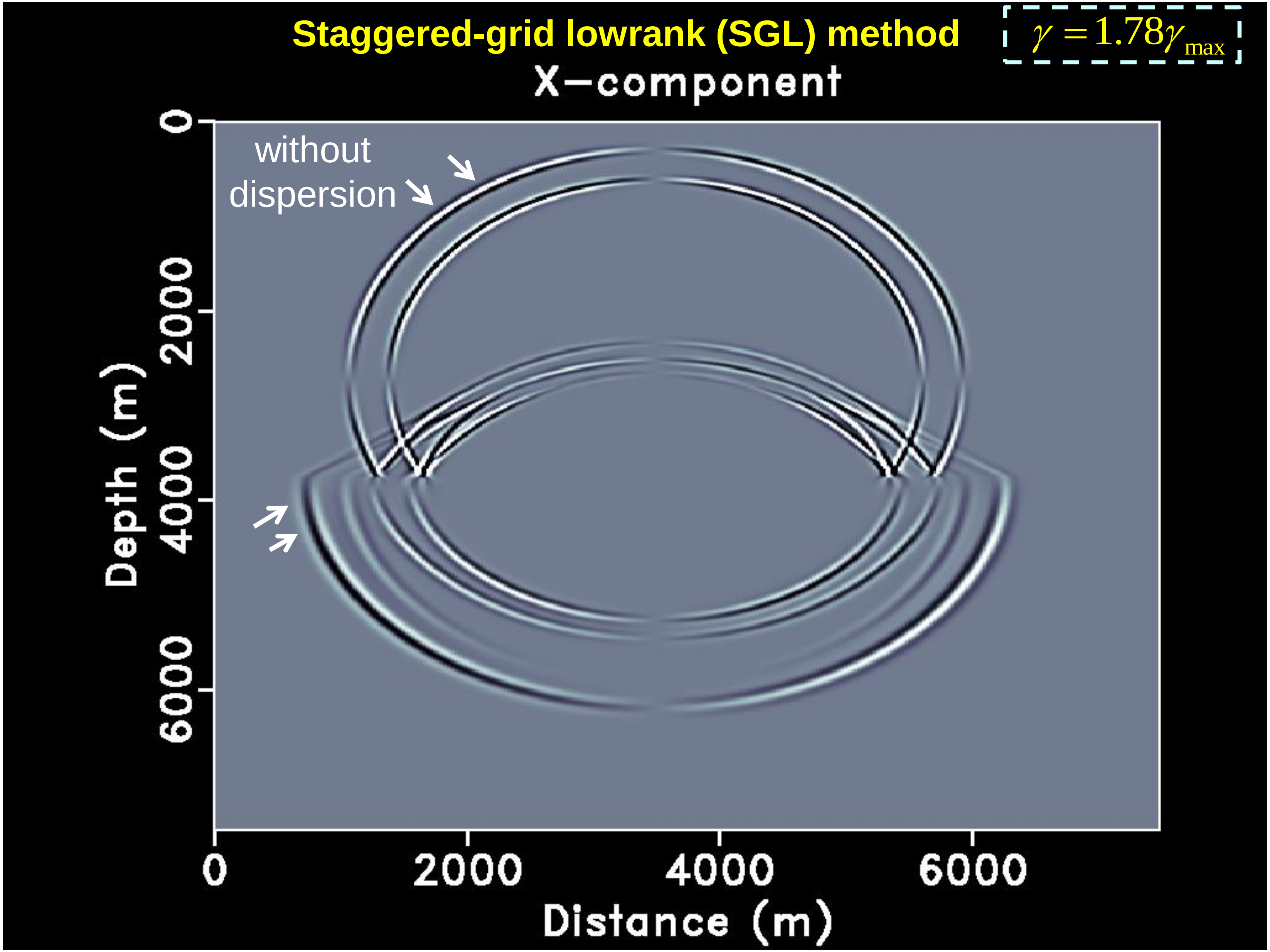
- Numerical simulation approach

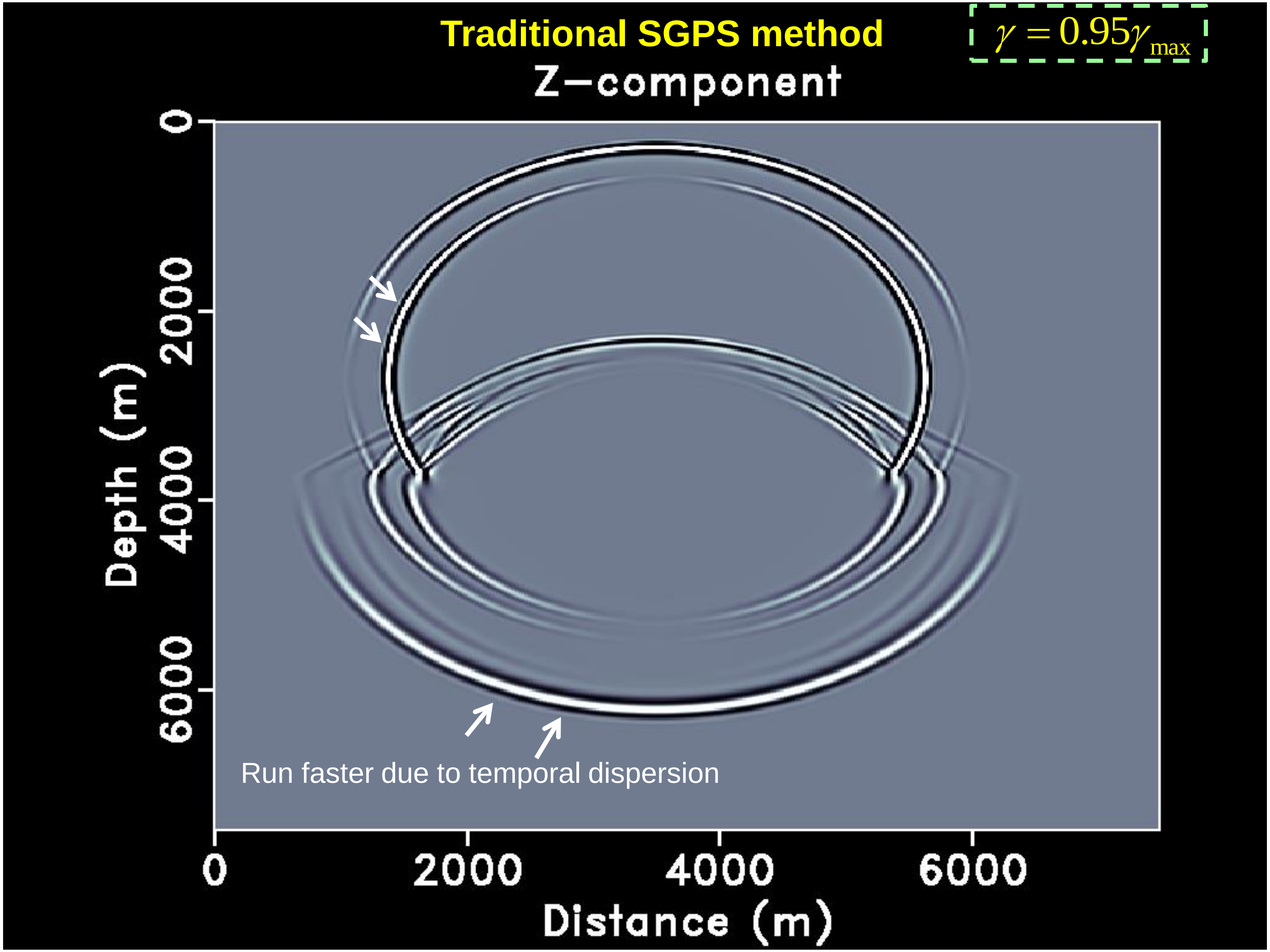
$$\begin{cases} \left(\dfrac{\partial \tau_{111}}{\partial x}\right)_{L_p} = IFFT\left\{(ik_x)e^{\pm i\frac{1}{2}k_x h}\,\mathrm{sinc}\left(\alpha\Delta t|k|/2\right)FFT\left(\tau_{111}\right)\right\}, \\ \left(\dfrac{\partial v_x}{\partial x}\right)_{L_s} = IFFT\left\{(ik_x)e^{\mp i\frac{1}{2}k_x h}\,\mathrm{sinc}\left(\beta\Delta t|k|/2\right)FFT\left(v_x\right)\right\}, \end{cases}$$

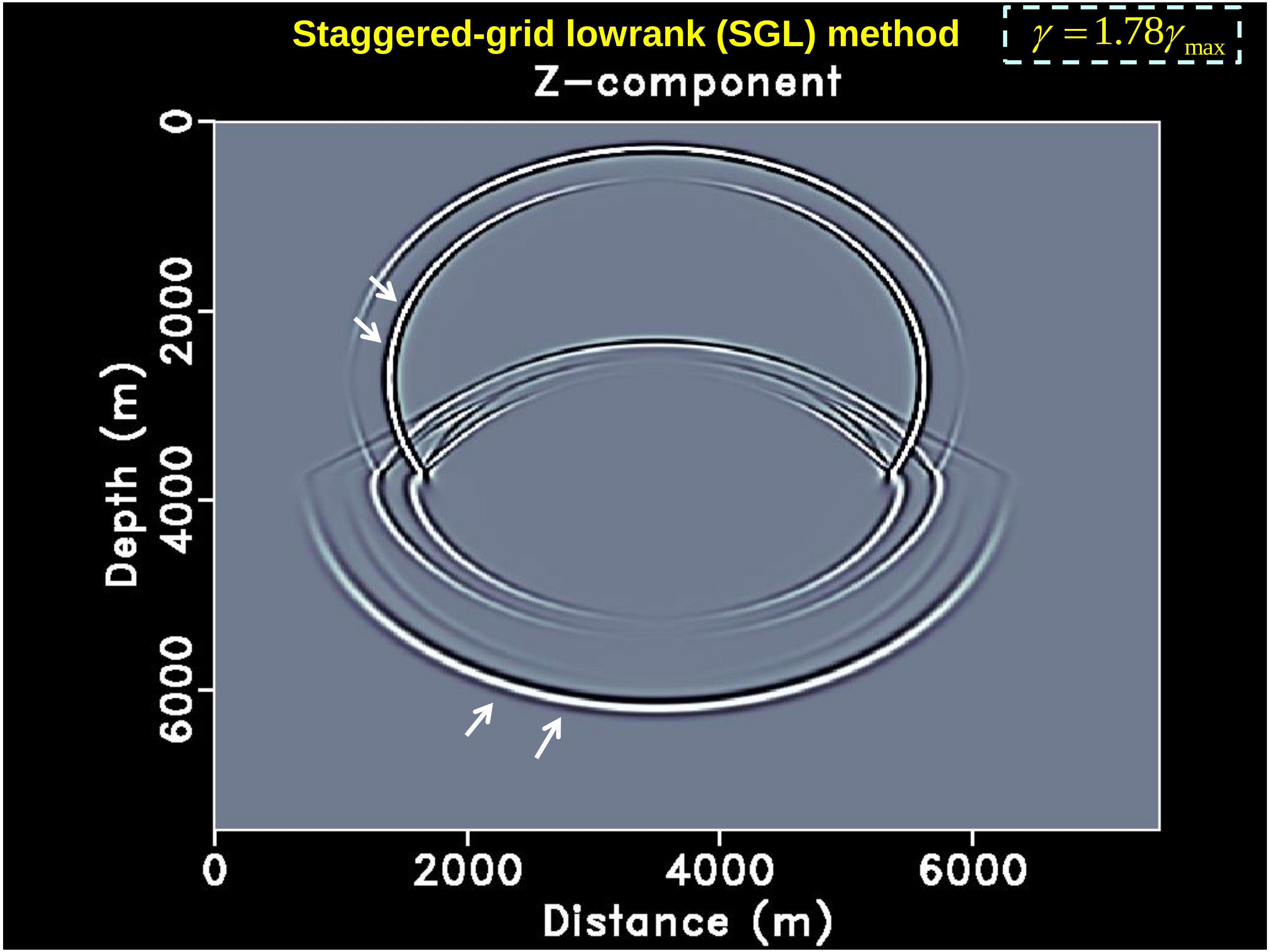
$$\alpha^2 = \frac{(\lambda+2\mu)}{\rho},\ \beta^2 = \frac{\mu}{\rho} \qquad \begin{cases} L_p = \mathrm{sinc}\left[\alpha(\mathbf{x})\Delta t|k|/2\right], \\ L_s = \mathrm{sinc}\left[\beta(\mathbf{x})\Delta t|k|/2\right]. \end{cases}$$

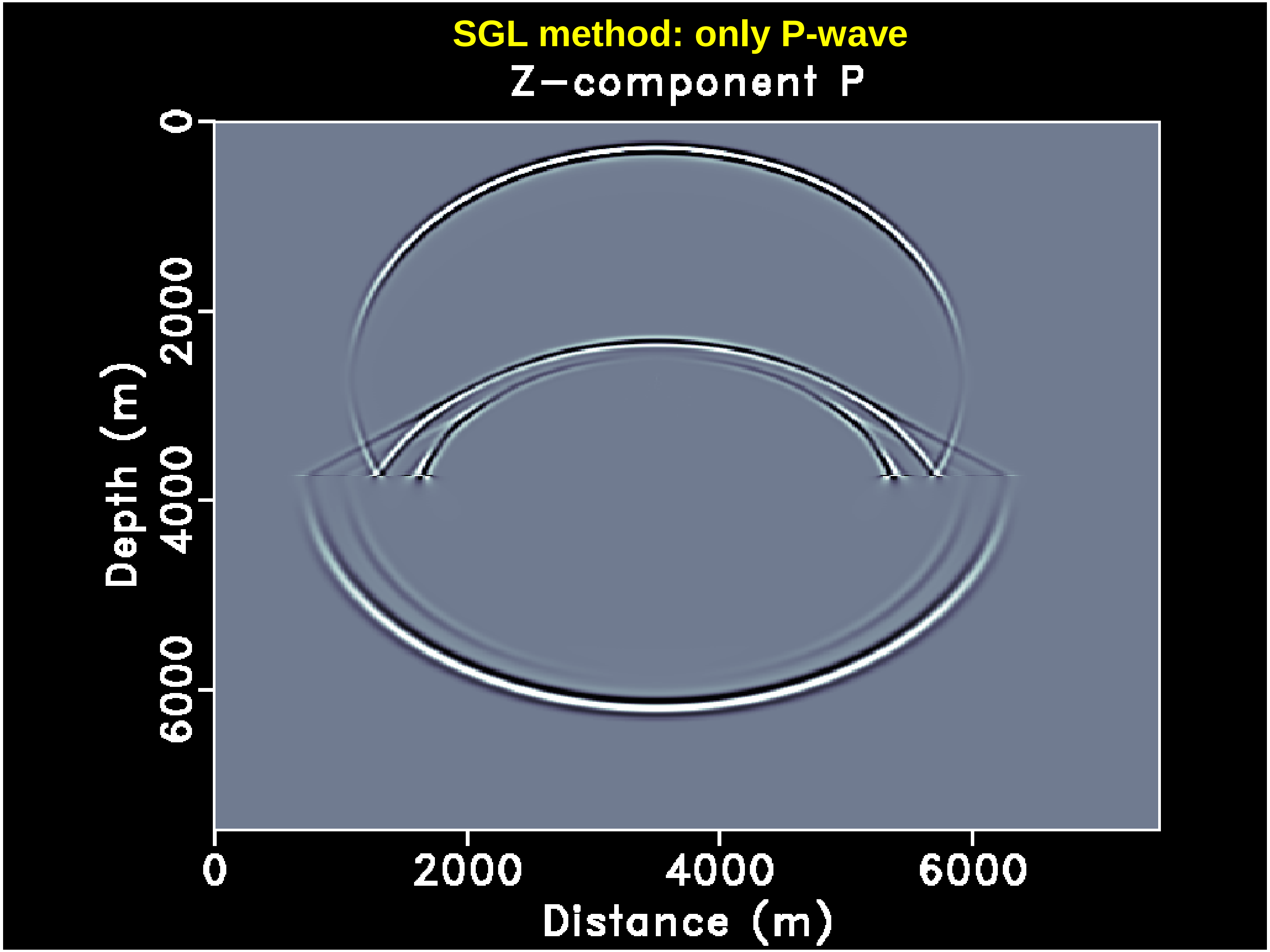
Two *k*-space operators related to P- and S-wave velocities respectively

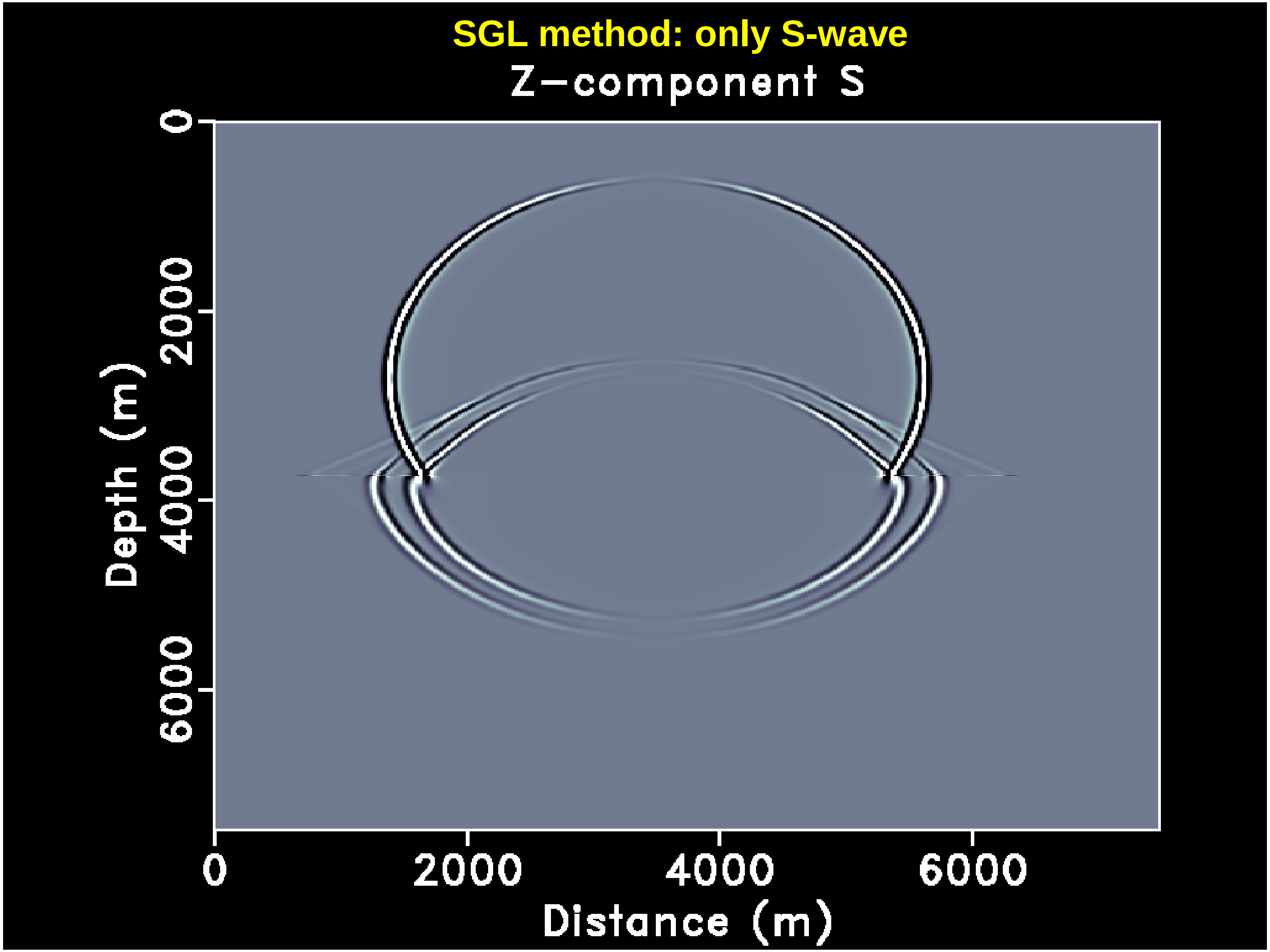

Traditional SGPS method
$\gamma = 0.95\gamma_{max}$
X-component
temporal
dispersion
Depth (m)
0
2000
4000
6000
Distance (m)
0
2000
4000
6000


Staggered-grid lowrank (SGL) method
$\gamma = 1.78\gamma_{max}$
X-component
without
dispersion
Depth (m)
Distance (m)
0
2000
4000
6000
0
2000
4000
6000

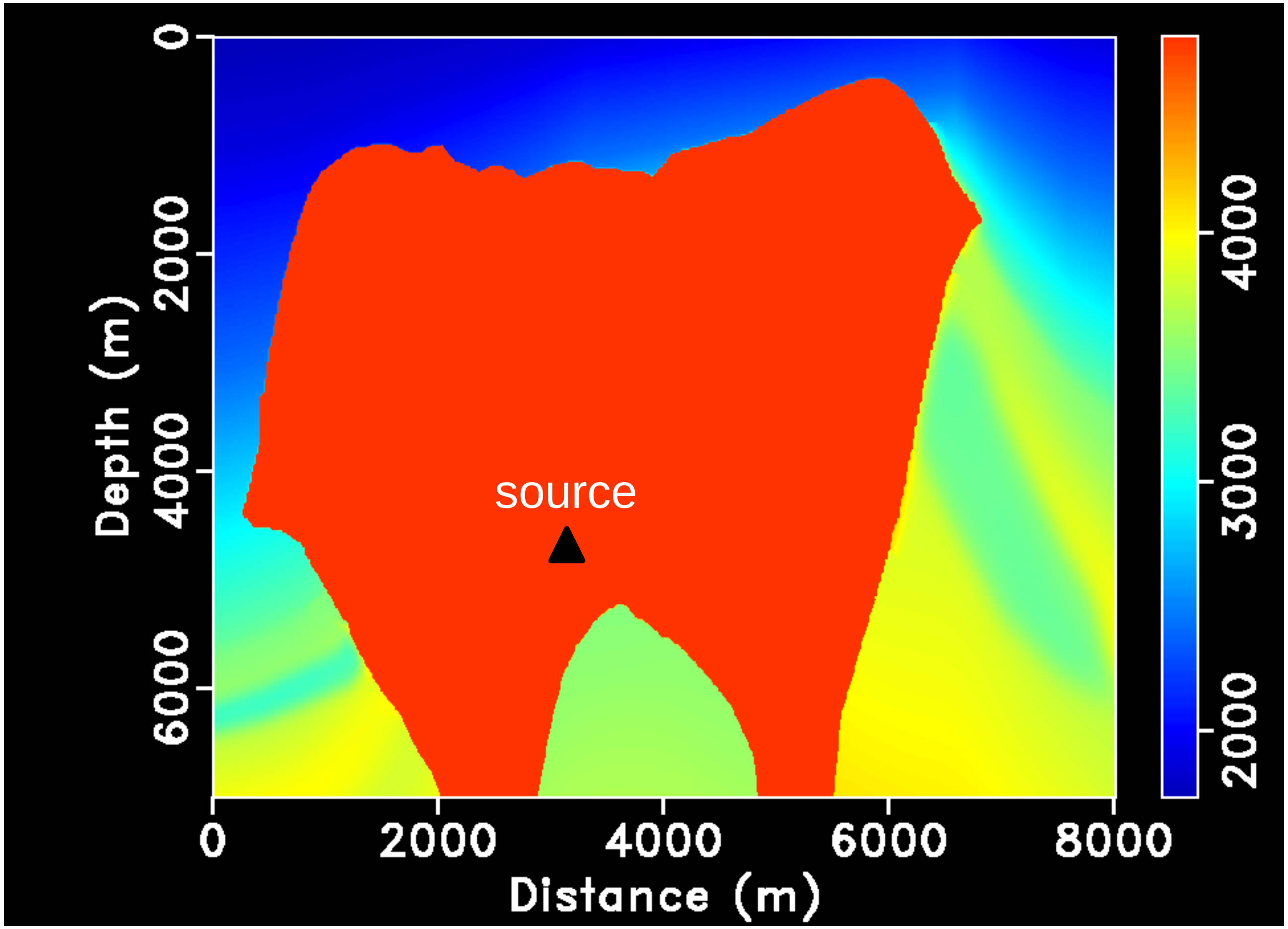

Traditional SGPS method
$\gamma = 0.95\gamma_{max}$
Z-component
Depth (m)
0
2000
4000
6000
Distance (m)
0
2000
4000
6000
Run faster due to temporal dispersion


Staggered-grid lowrank (SGL) method
$\gamma = 1.78\gamma_{max}$
Z-component
Depth (m)
0
2000
4000
6000
Distance (m)
0
2000
4000
6000


SGL method: only P-wave
Z-component P
Depth (m)
0
2000
4000
6000
Distance (m)
0
2000
4000
6000

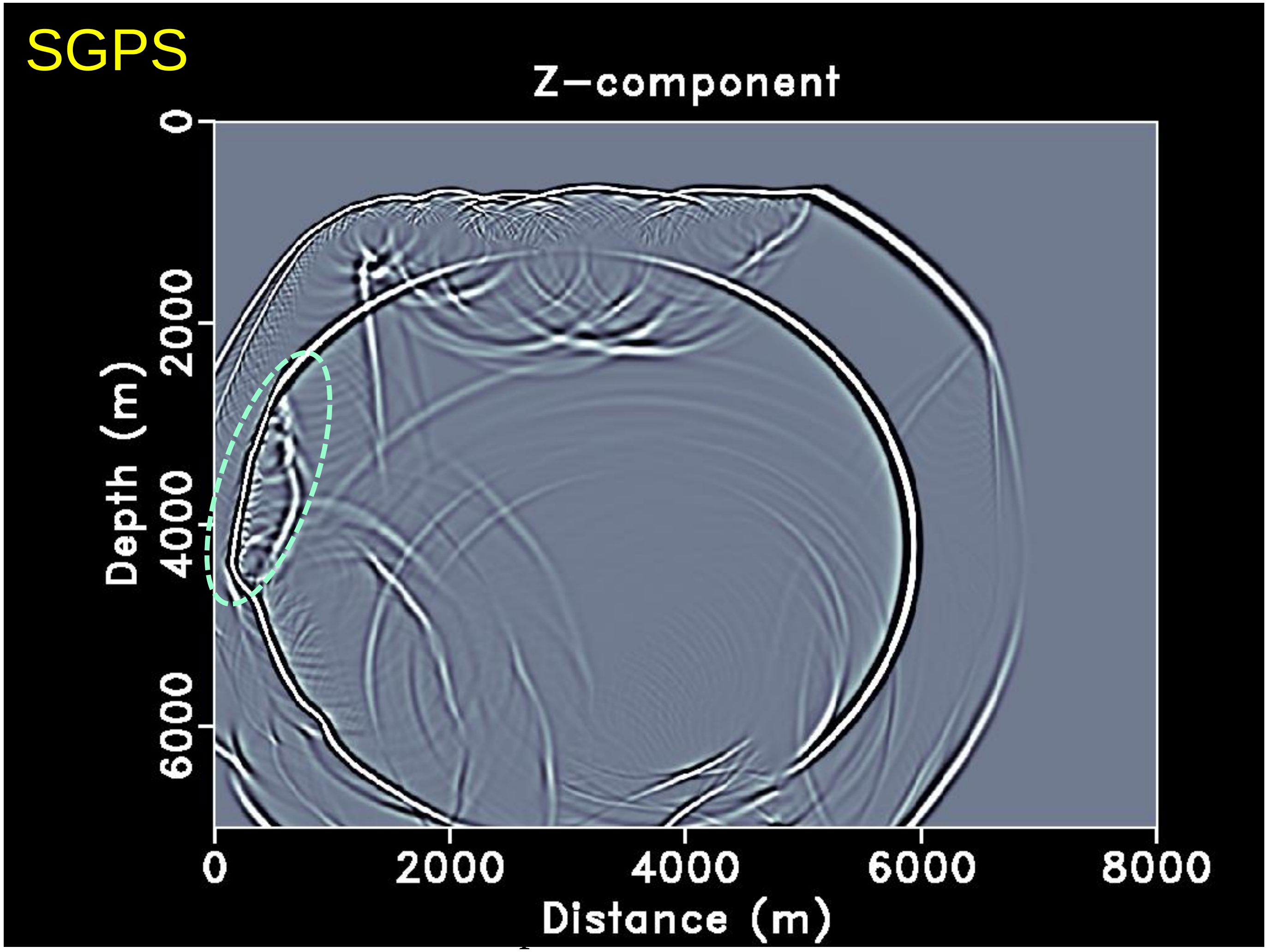

SGL method: only S-wave
Z-component S
Depth (m)
0
2000
4000
6000
Distance (m)
0
2000
4000
6000

Elastic lowrank extrapolation

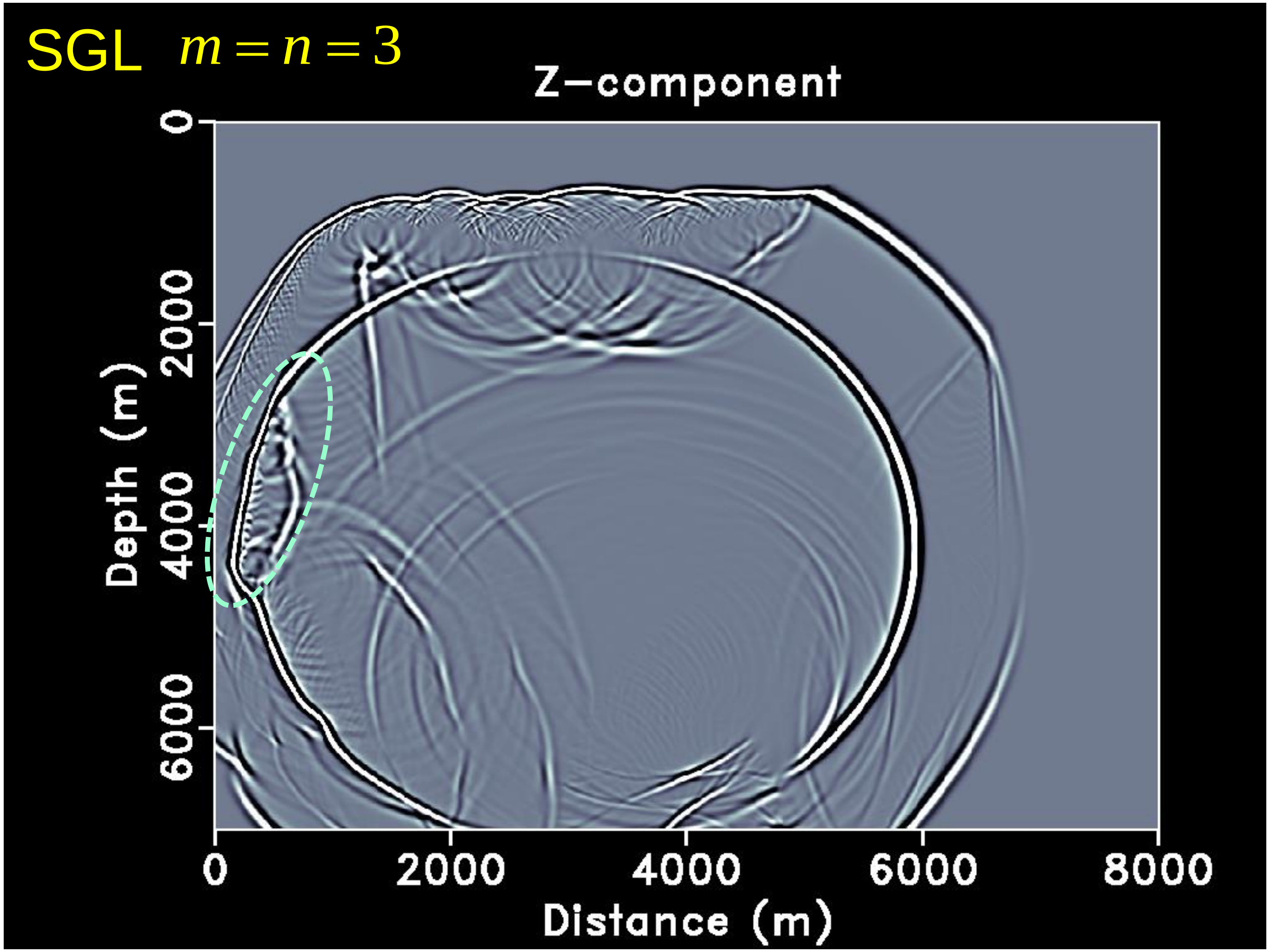


$$\beta = \frac{\alpha}{1.4}$$

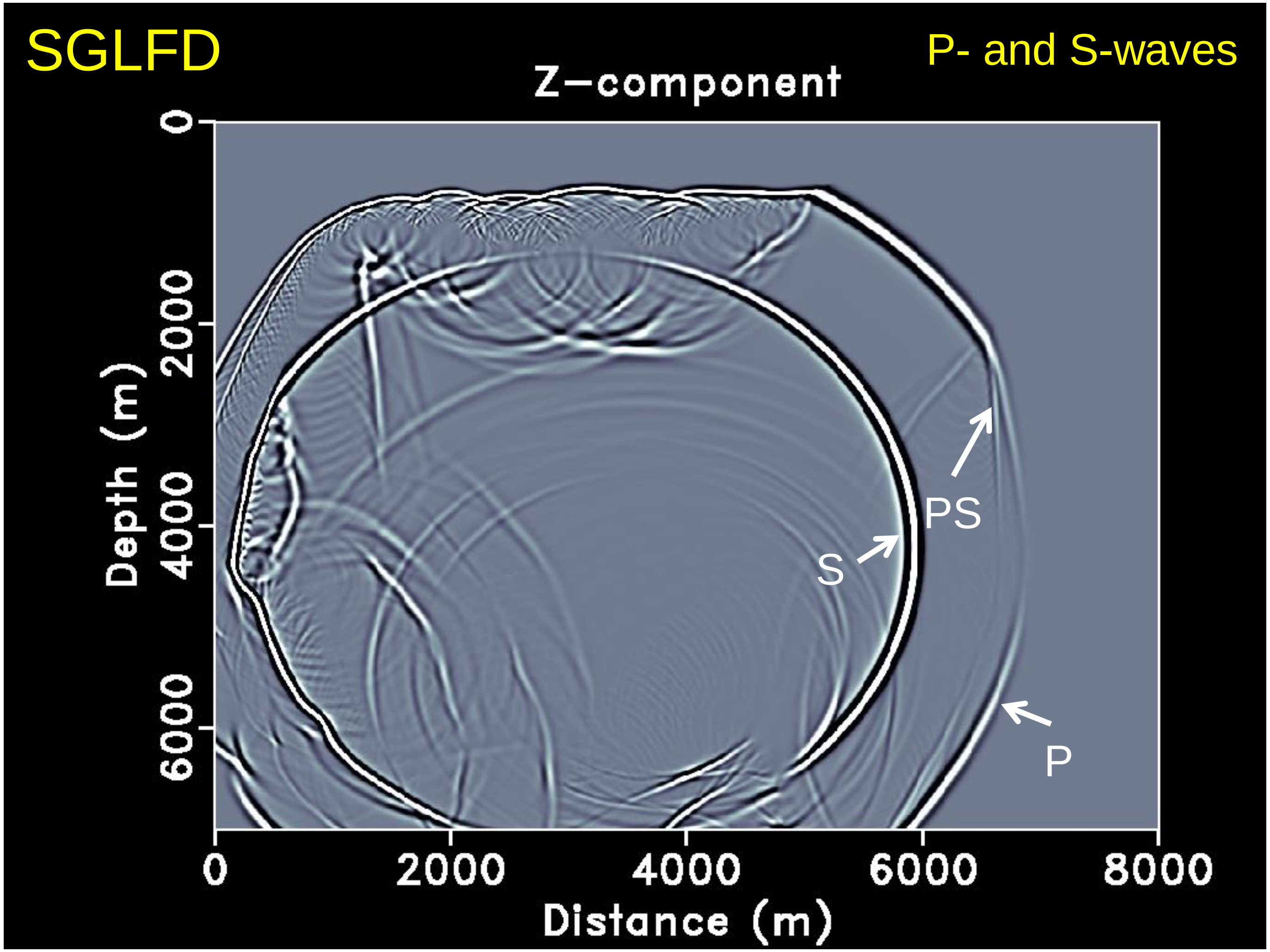
Elastic lowrank extrapolation



Elastic lowrank extrapolation

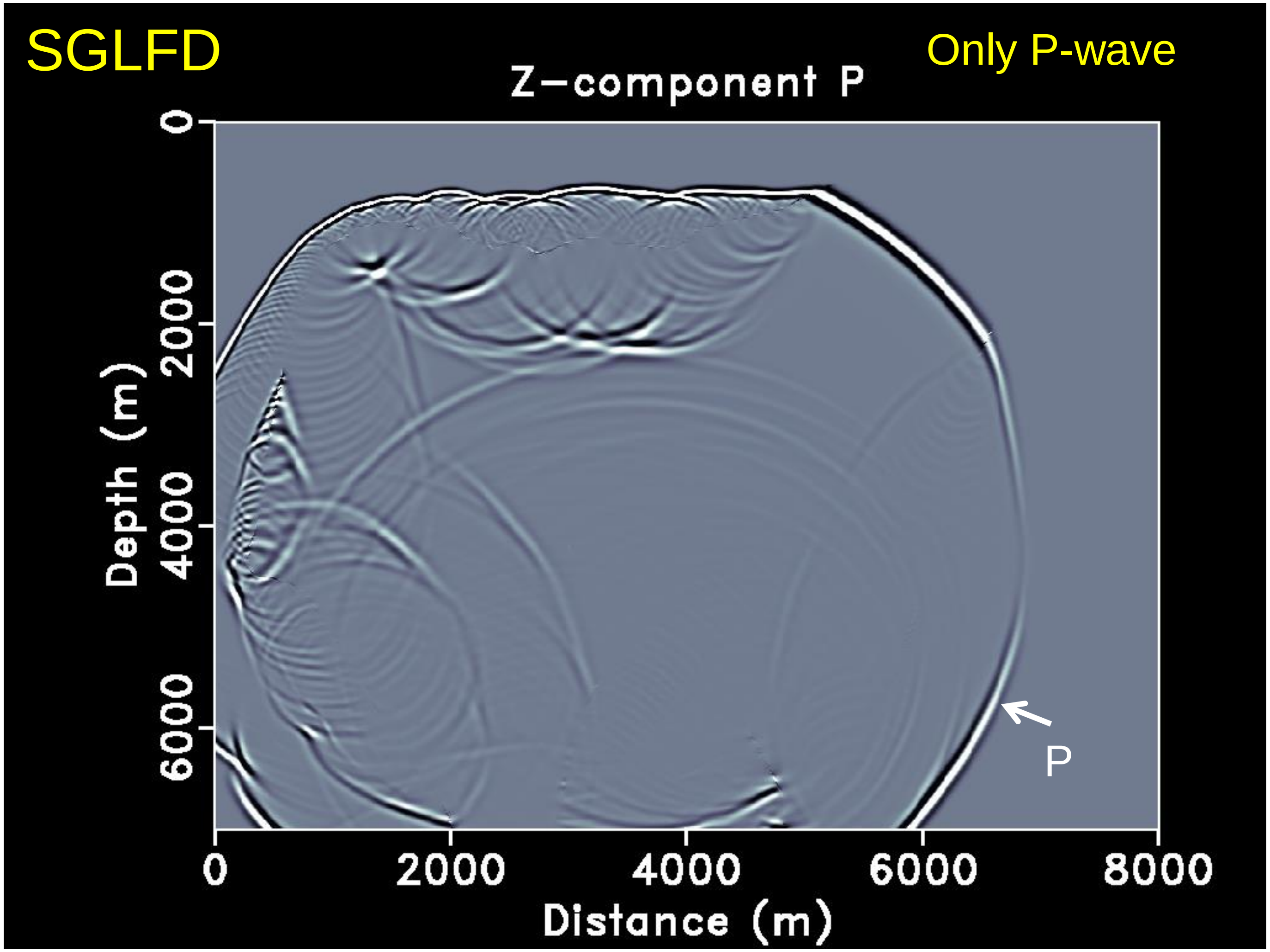


Elastic lowrank extrapolation

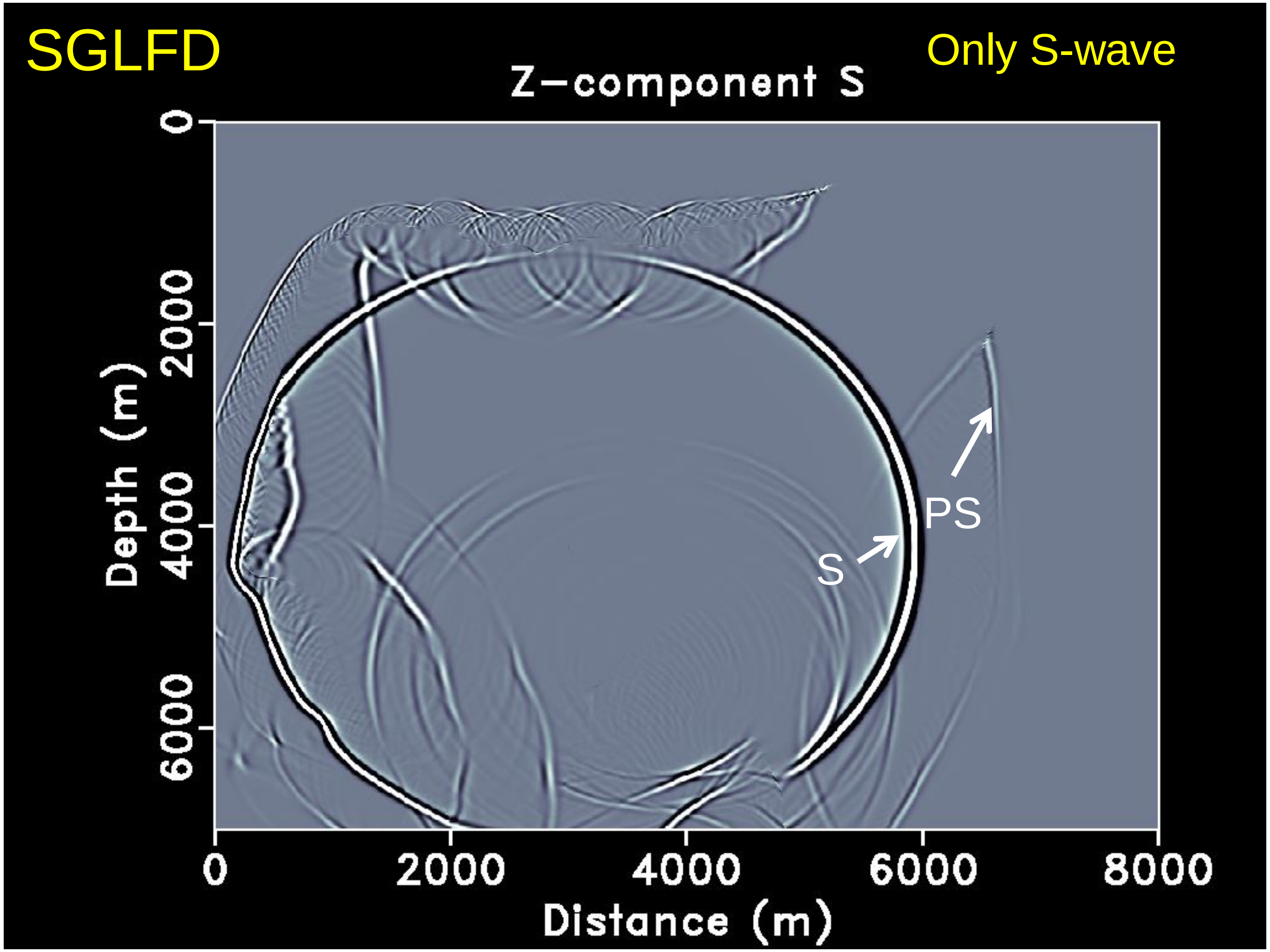


stencil length $2M_p = 8,\ 2M_s = 12$

Elastic lowrank extrapolation



Elastic lowrank extrapolation



Viscoacoustic lowrank extrapolation

- Fractional Laplacian viscoacoustic wave equation (T. Zhu and J. Harris, 2014)

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = \eta\left(-\nabla^2\right)^{\gamma+1} p + \tau\frac{d}{dt}\left(-\nabla^2\right)^{\gamma+\frac{1}{2}} p$$

$$\begin{cases} \eta = -c_o^{2\gamma}\omega_o^{-2\gamma}\cos(\pi\gamma), \\ \tau = -c_o^{2\gamma-1}\omega_o^{-2\gamma}\sin(\pi\gamma), \\ c^2 = c_o^2\cos^2(\pi\gamma/2), \\ \gamma = 1/\pi\tan^{-1}(1/Q). \end{cases} \quad \xrightarrow{Q\to\infty} \quad \begin{cases} \eta = -1, \\ \tau = 0, \\ c^2 = c_o^2 \\ \gamma = 0. \end{cases}$$

Degrades to

$$\frac{1}{c_o^2}\frac{\partial^2 p}{\partial t^2} = \nabla^2 p$$

Major advantages:

a. high accuracy for approximating constant-*Q* model

b. decoupled dispersion and attenuation

Viscoacoustic lowrank extrapolation

- Exact time marching scheme

$$\frac{d^2\tilde{p}}{dt^2} - c^2\tau|\mathbf{k}|^{2\gamma+1}\frac{d\tilde{p}}{dt} - c^2\eta|\mathbf{k}|^{2\gamma+2}\,\tilde{p} = 0$$

Analytical solution

$$\tilde{p}(t) = e^{-\alpha t}\left[A\cos(\beta t) + B\sin(\beta t)\right]$$

⬇

$$e^{\alpha(t+\Delta t)}\tilde{p}(t+\Delta t) + e^{\alpha(t-\Delta t)}\tilde{p}(t-\Delta t) = 2e^{\alpha t}\cos(\beta\Delta t)\,\tilde{p}(t)$$

$$\begin{cases} \alpha = \overline{\tau}|\mathbf{k}|^{2\gamma+1},\ \beta = \overline{\eta}|\mathbf{k}|^{\gamma+1}, \\ \overline{\tau} = \frac{1}{2}c_o^{2\gamma+1}\omega_o^{-2\gamma}\cos^2\left(\frac{\pi\gamma}{2}\right)\sin(\pi\gamma), \\ \overline{\eta} = \frac{1}{2}c_o^{\gamma+1}\omega_o^{-\gamma}\cos\left(\frac{\pi\gamma}{2}\right)\left[4\cos(\pi\gamma) - \cos^2\left(\frac{\pi\gamma}{2}\right)\sin^2(\pi\gamma)\right]^{\frac{1}{2}}. \end{cases}$$

Viscoacoustic lowrank extrapolation

- Exact time marching scheme

$$\tilde{p}(\mathbf{k}, t+\Delta t) = 2e^{-\alpha\Delta t}\cos(\beta\Delta t)\,\tilde{p}(\mathbf{k}, t) - e^{-2\alpha\Delta t}\tilde{p}(\mathbf{k}, t-\Delta t)$$

$$p(\mathbf{x}, t+\Delta t) = 2F^{-1}\left\{e^{-\alpha\Delta t}\cos(\beta\Delta t)\,F\left[p(\mathbf{x}, t)\right]\right\} - F^{-1}\left\{F\left[e^{-2\alpha\Delta t}p(\mathbf{x}, t-\Delta t)\right]\right\}$$

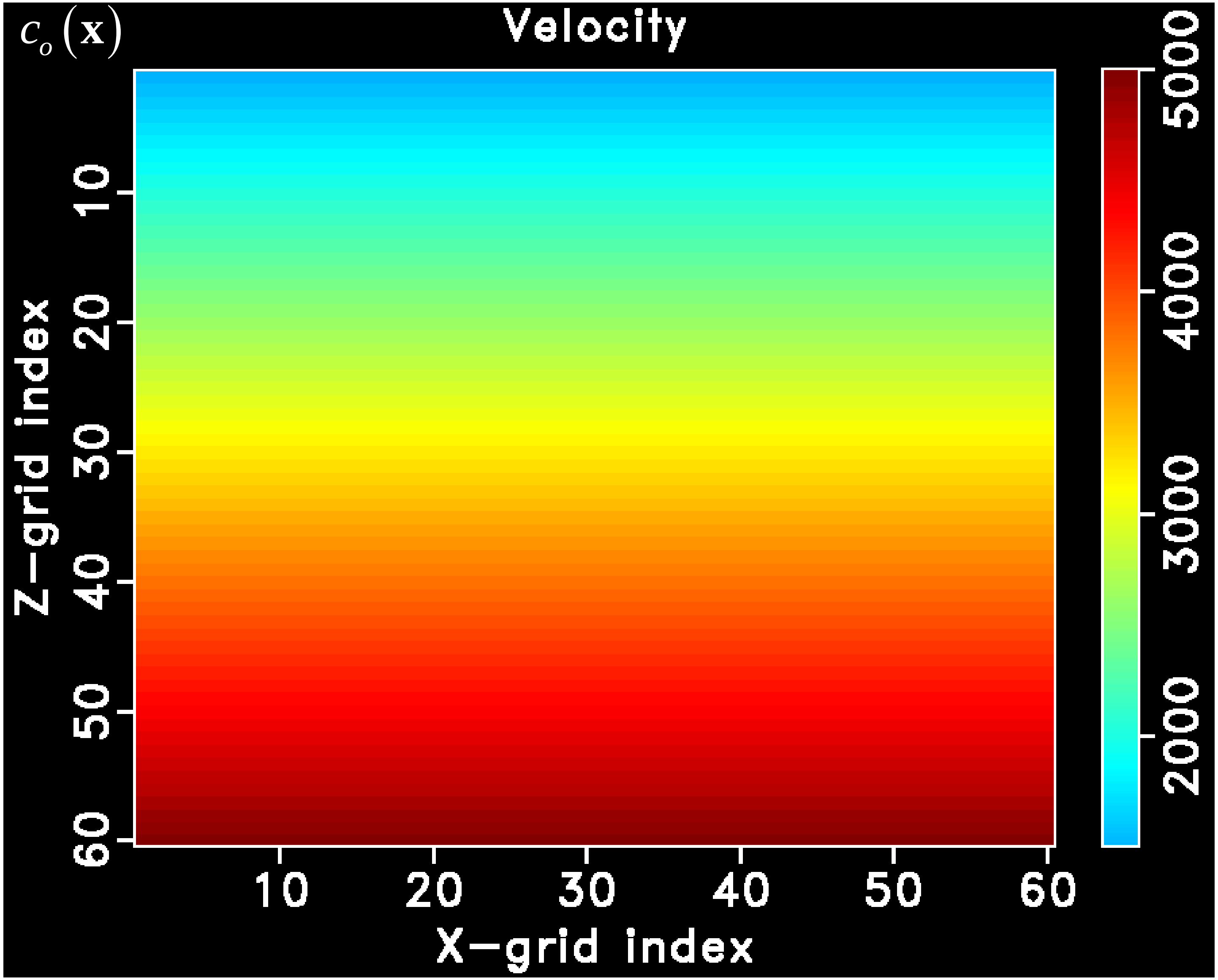
Two mixed domain operators

$$\mathbf{W}_1(\mathbf{x},\ k) = e^{-\alpha\Delta t}\cos(\beta\Delta t) = \mathbf{U}_1(\mathbf{x}, \mathbf{k}_m)_{N\times m} \times \mathbf{A}_{1m\times n} \times \mathbf{V}_1(\mathbf{x}_n, \mathbf{k})_{n\times N}$$

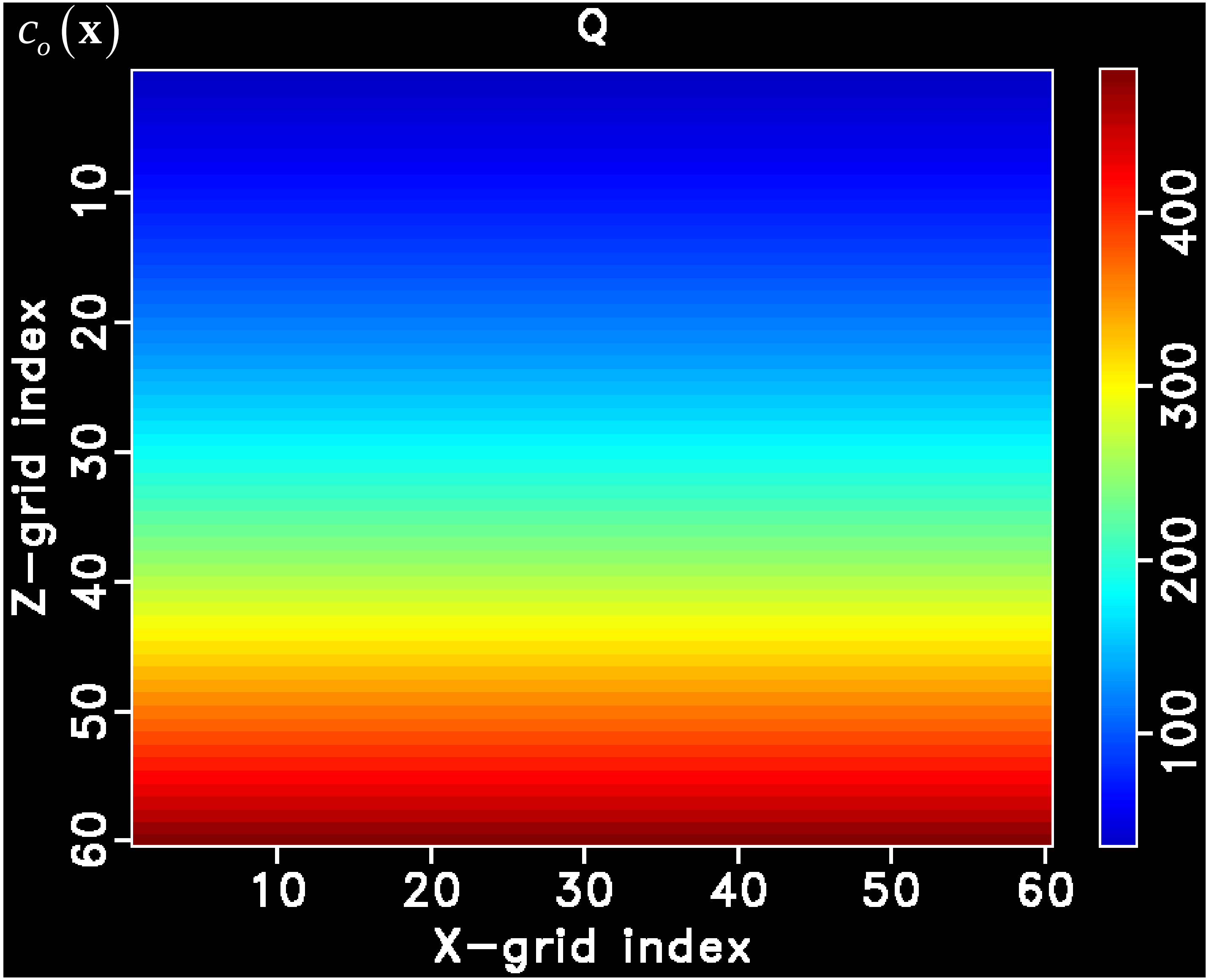
$$\mathbf{W}_2(\mathbf{x},\ k) = e^{-2\alpha\Delta t} = \mathbf{U}_2(\mathbf{x}, \mathbf{k}_m)_{N\times m} \times \mathbf{A}_{2m\times n} \times \mathbf{V}_2(\mathbf{x}_n, \mathbf{k})_{n\times N}$$

$$c_o(\mathbf{x}),\ Q(\mathbf{x})$$

Viscoacoustic lowrank extrapolation

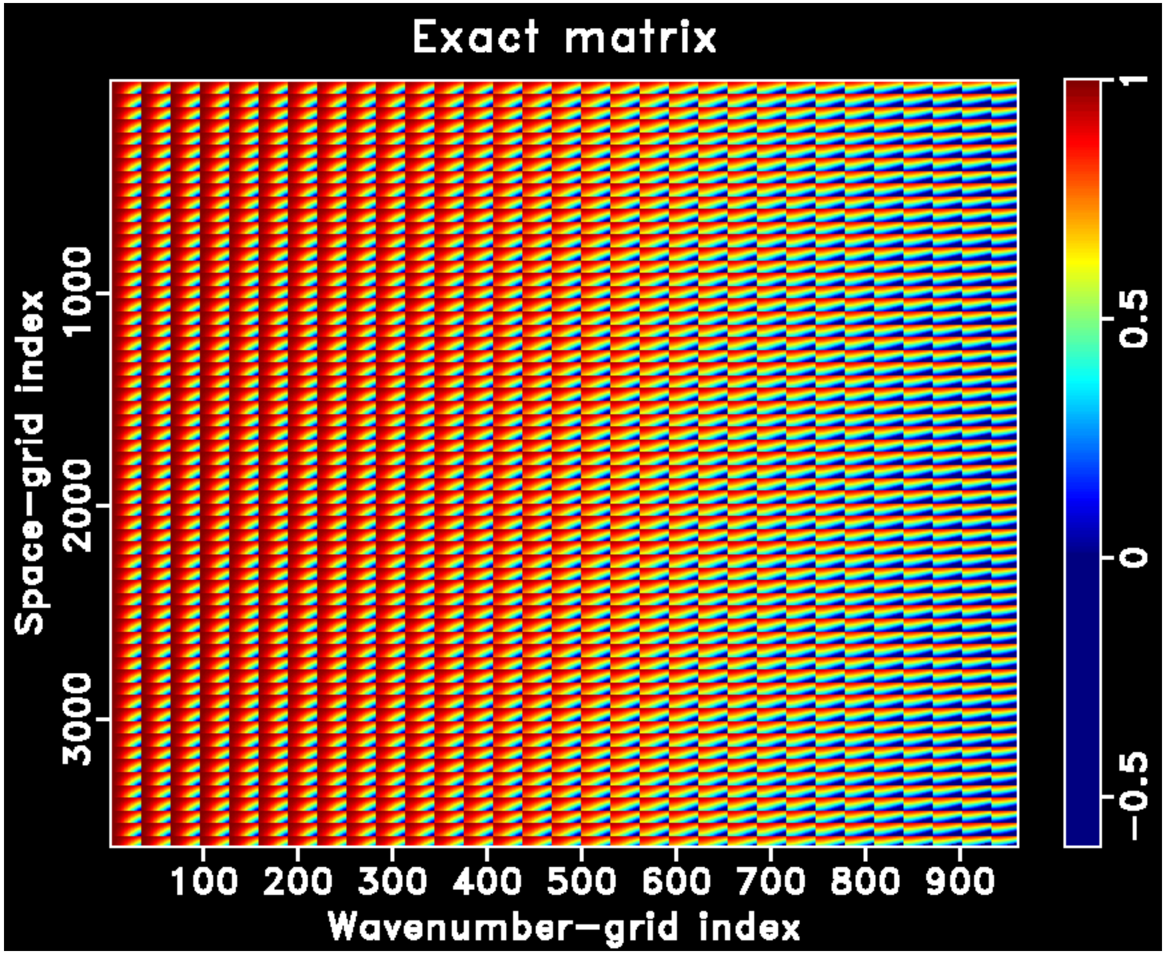


Viscoacoustic lowrank extrapolation



Viscoacoustic lowrank extrapolation

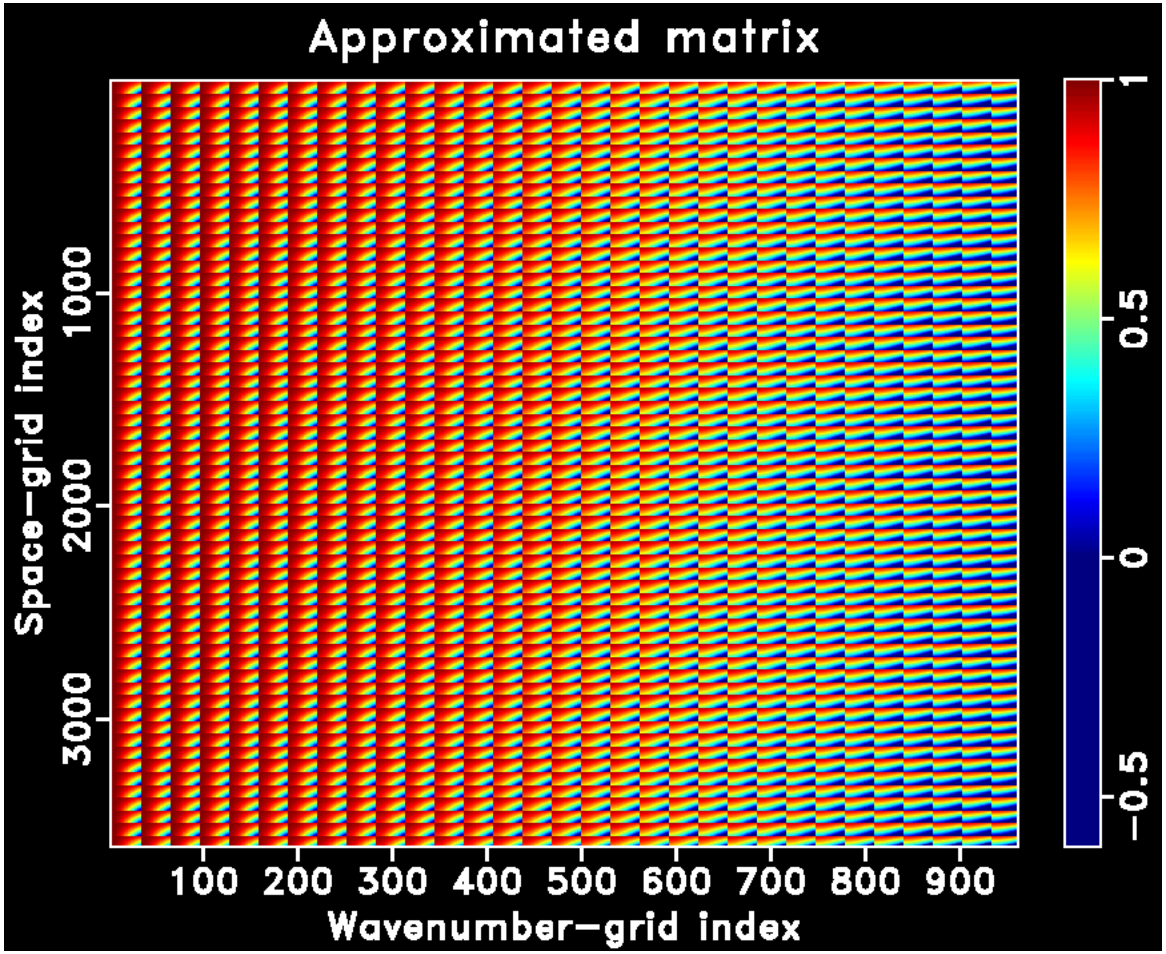
- Viscoacoustic lowrank decomposition test (by my C program)



$$\mathbf{W}_1(\mathbf{x},\ k) = e^{-\alpha \Delta t}\cos(\beta \Delta t)$$

Viscoacoustic lowrank extrapolation

- Viscoacoustic lowrank decomposition test (by my C program)

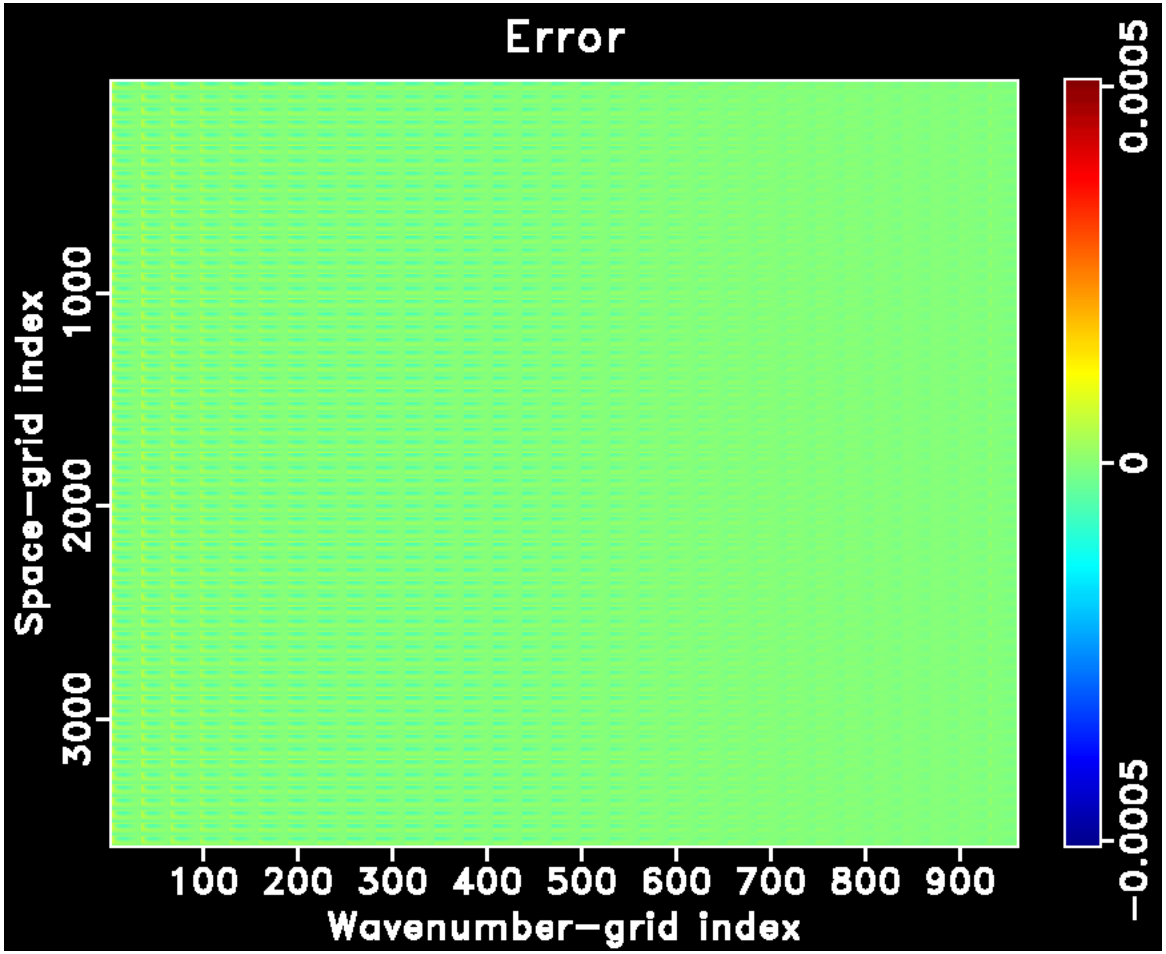


$m = n = 4$

$$\mathbf{W}_1(\mathbf{x},\ k) = \mathbf{U}_1(\mathbf{x},\mathbf{k}_m)_{N\times m} \times \mathbf{A}_{1m\times n} \times \mathbf{V}_1(\mathbf{x}_n,\mathbf{k})_{n\times N}$$

Viscoacoustic lowrank extrapolation

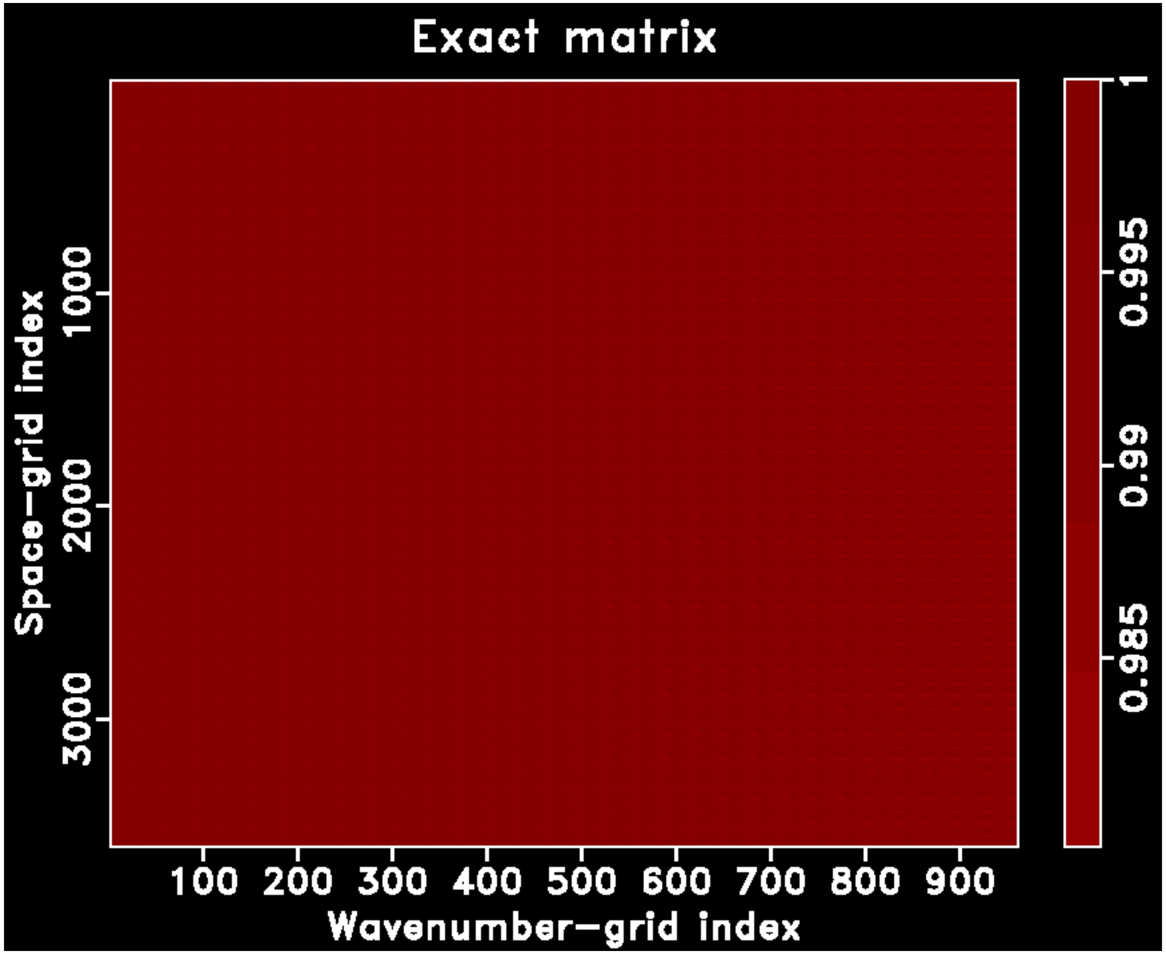
- Viscoacoustic lowrank decomposition test (by my C program)



$m = n = 4$

Viscoacoustic lowrank extrapolation

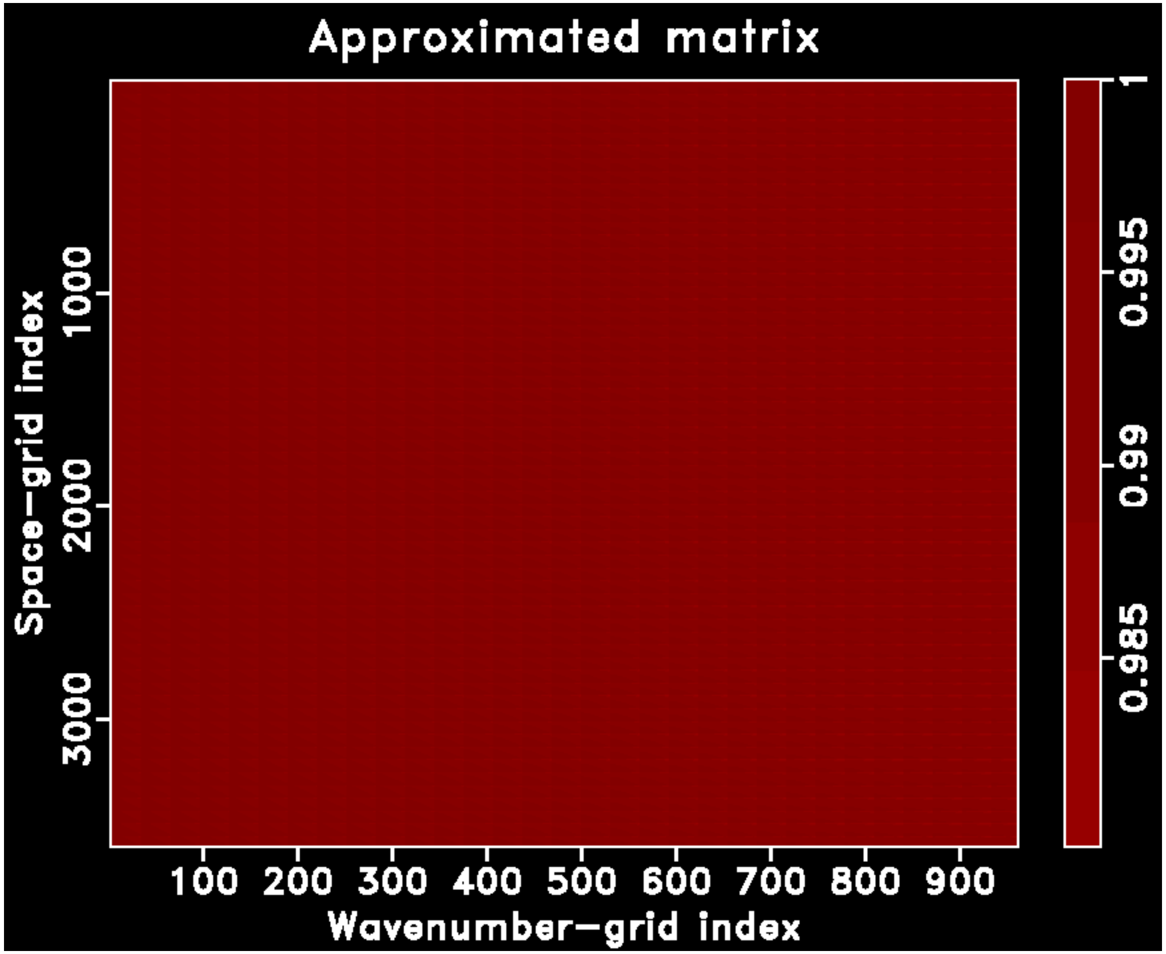
- Viscoacoustic lowrank decomposition test (by my C program)



$$\mathbf{W}_2(\mathbf{x},\ k) = e^{-2\alpha\Delta t}$$

Viscoacoustic lowrank extrapolation

- Viscoacoustic lowrank decomposition test (by my C program)

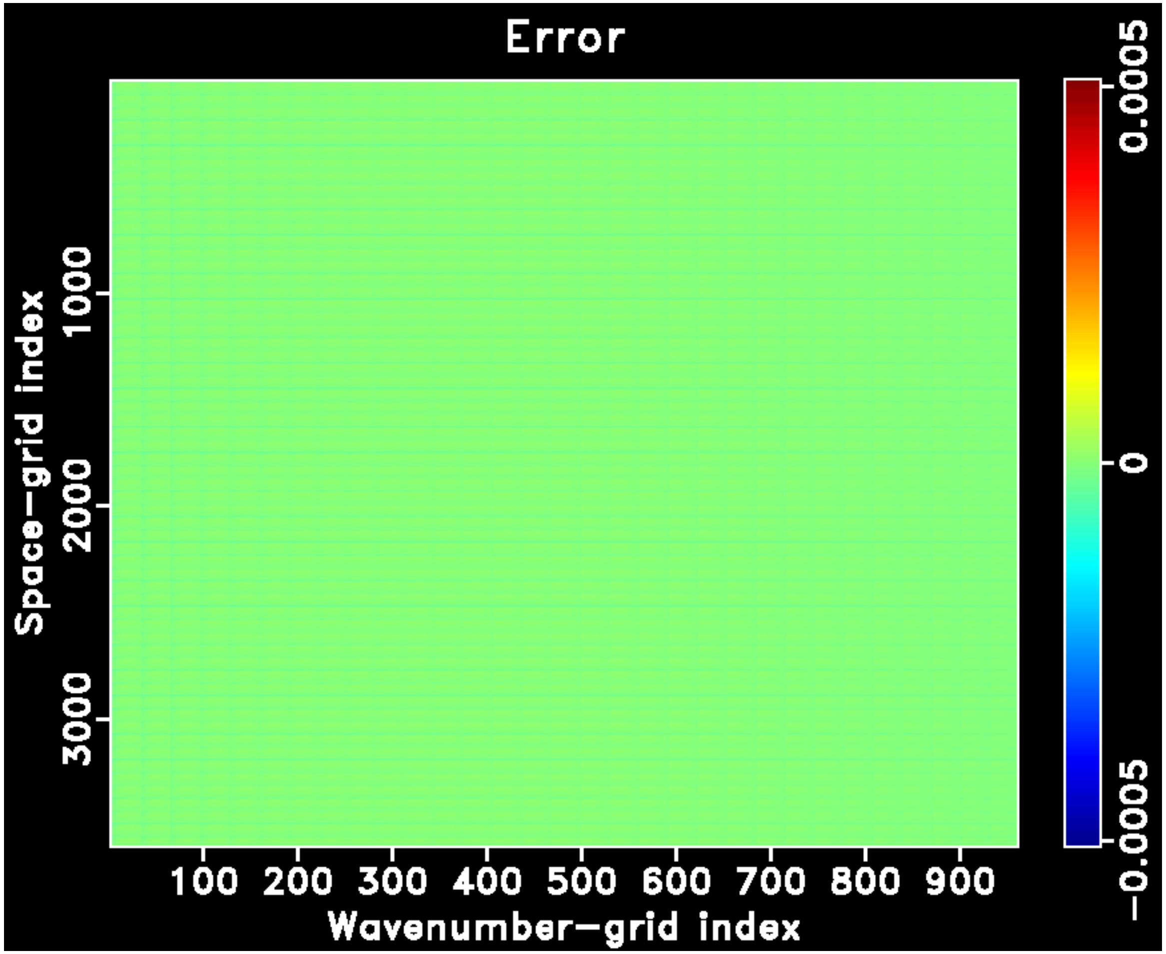


$m = n = 2$

$$\mathbf{W}_2(\mathbf{x},\ k) = \mathbf{U}_2(\mathbf{x},\mathbf{k}_m)_{N\times m} \times \mathbf{A}_{2m\times n} \times \mathbf{V}_2(\mathbf{x}_n,\mathbf{k})_{n\times N}$$

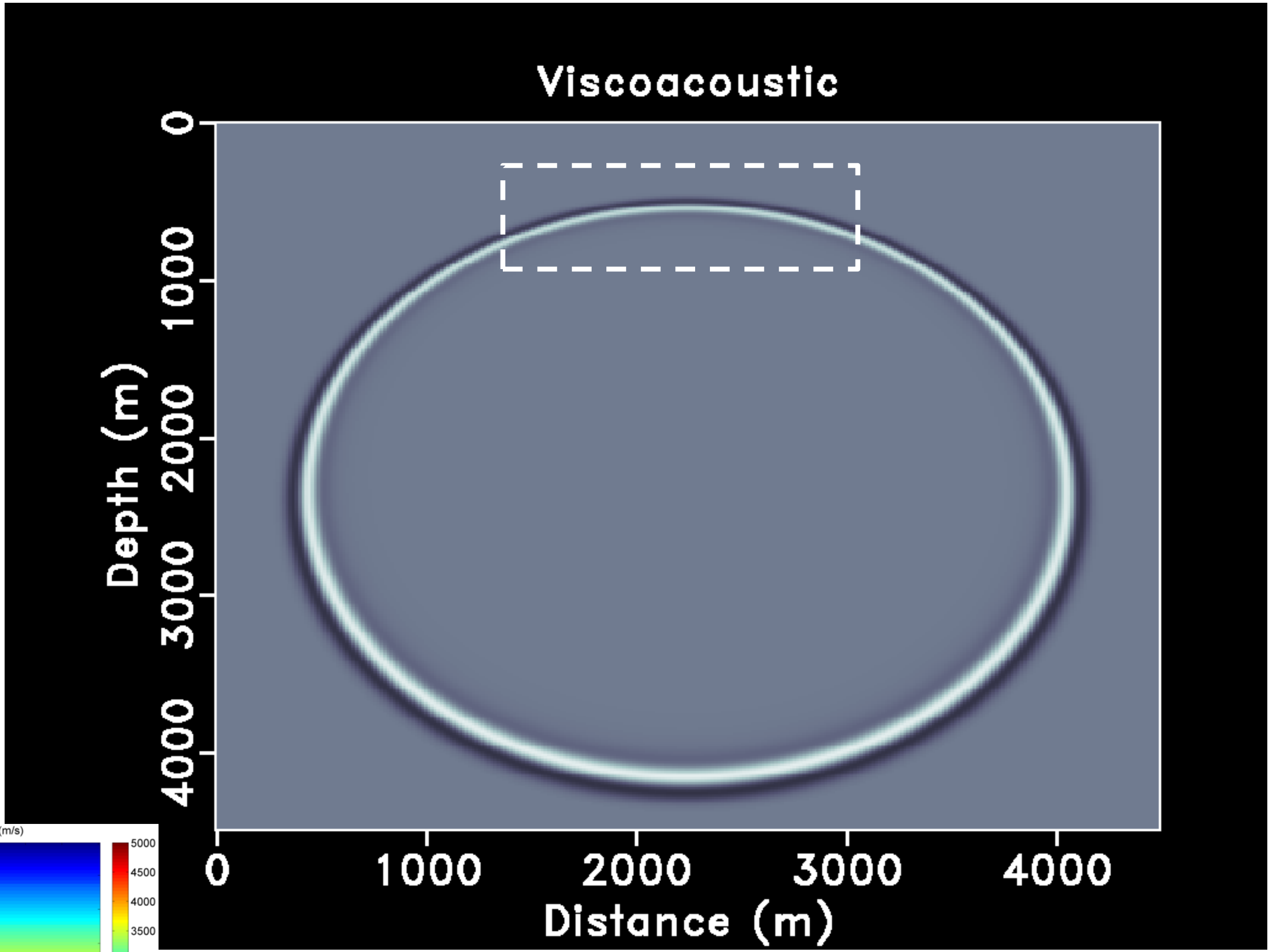
Viscoacoustic lowrank extrapolation

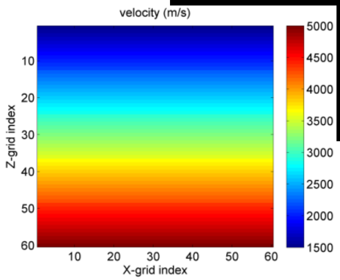
- Viscoacoustic lowrank decomposition test (by my C program)



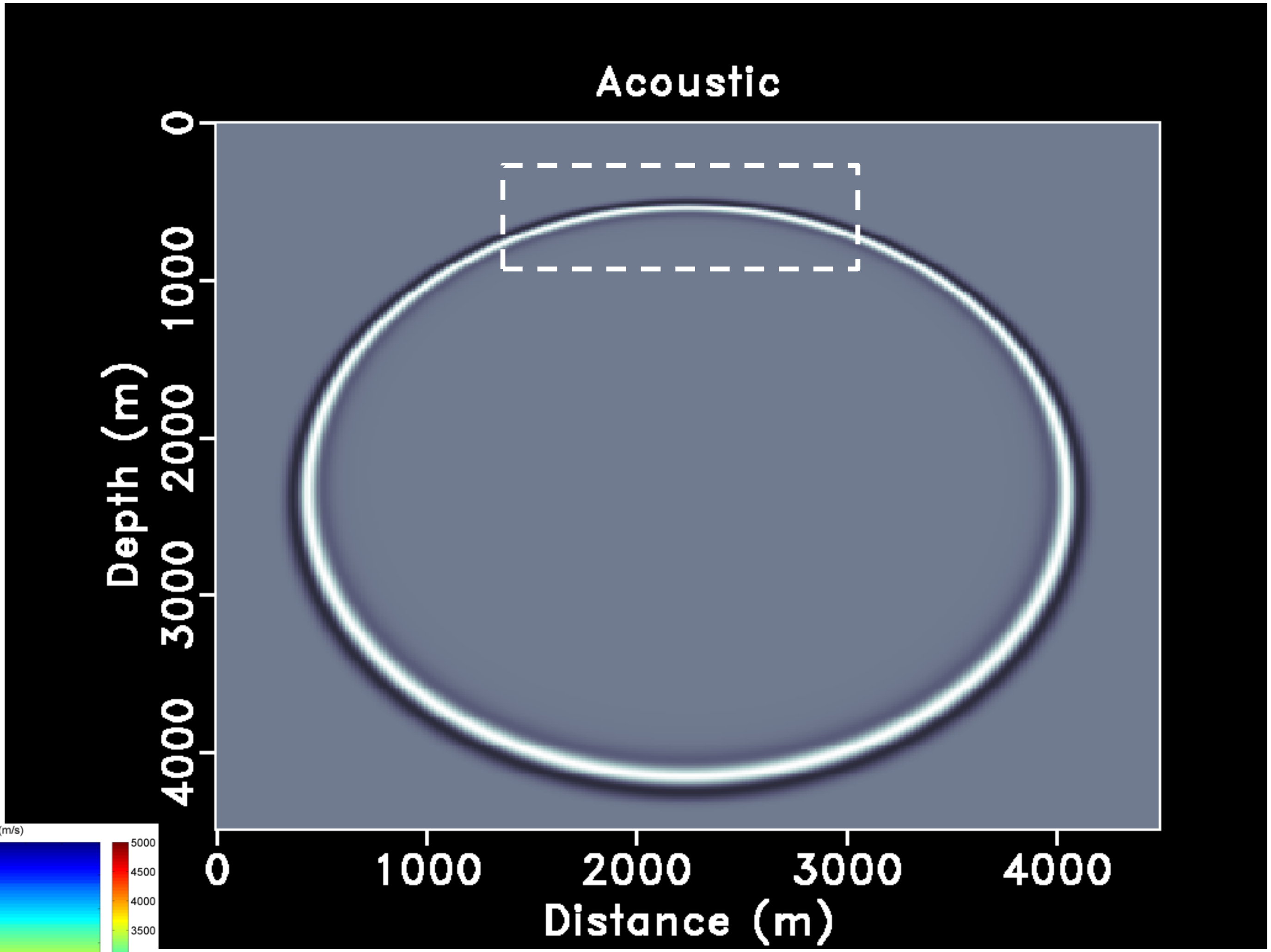
$m = n = 2$

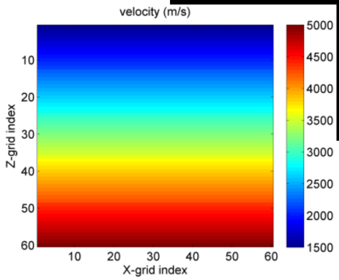
Viscoacoustic lowrank extrapolation



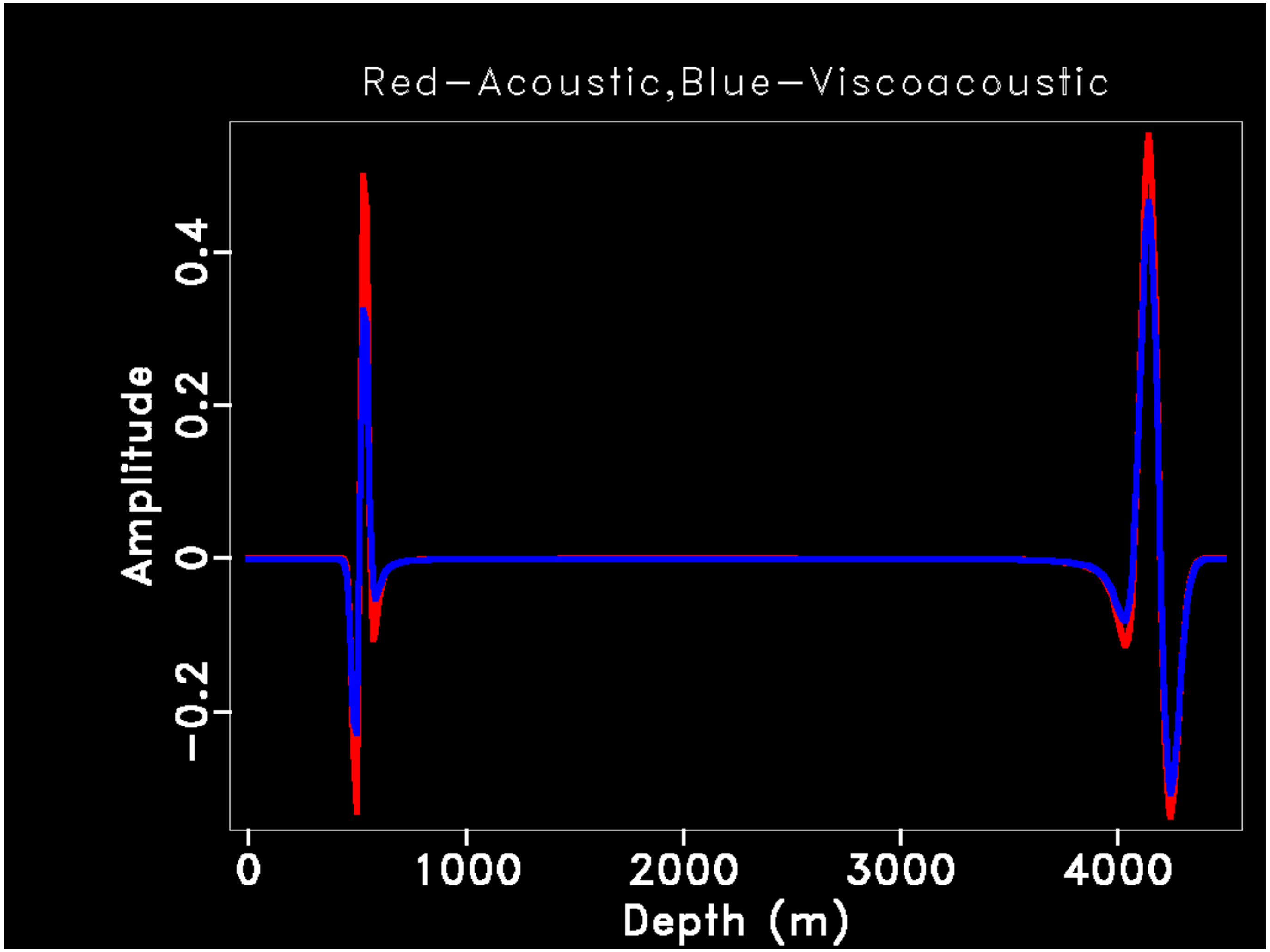


Viscoacoustic lowrank extrapolation





Viscoacoustic lowrank extrapolation



Conclusions

- Lowrank decomposition is an effective tool for seismic wave modeling that can be applied in acoustic, elastic, viscoacoustic and even more complicated wave equations.
- I strongly feel that Madagascar software can make me run more efficiently.

Gratitude

I appreciate the Graduate Research Assistant Yangkang Chen at TCCS for teaching me Madagascar operation and helping me to transform my C code into Madagascar programs in the past three weeks.

Thanks for your attention!