Wavefield Seismic Imaging tutorial:

"exploding reflector" modeling/migration

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ABSTRACT

This document demonstrates how numeric examples constructed using the MADAGASCAR software package can be integrated into a reproducible document generated using the LaTeX typesetting program. I use a simple modeling/migration exercise based on the exploding reflector model to illustrate the main features of this process.

INTRODUCTION

The exploding reflector model, illustrated in Figure 1, allows us to perform zero offset modeling and migration for models of arbitrary complexity (Clærbout, 1985). Under this model, the image is described as a collection of points which "explode", i.e. become sources, at the same time arbitrarily set to be the time origin. Data are obtained at the receivers by forward simulation of acoustic waves from the exploding reflectors. Likewise, images are obtained by backward simulation of acoustic waves from the observed data.

Acoustic modeling and migration can be implemented using numeric solutions to an acoustic wave-equation, for example a variable-density wave-equation:

$$\frac{1}{v^2}\frac{\partial^2 W}{\partial t^2} - \rho\nabla\cdot\left(\frac{1}{\rho}\nabla W\right) = f \ . \tag{1}$$

In Equation 1, $W(\mathbf{x},t)$ represents the acoustic wavefield, $v(\mathbf{x})$ and $\rho(\mathbf{x})$ represent the velocity and density of the medium, respectively, and $f(\mathbf{x},t)$ represents a source function.

- In **modeling**, we use the distributed source $f(\mathbf{x},t)$ to generate the wavefield $W(\mathbf{x},t)$ at all positions and all times by wave propagation forward in time. The data represent a subset of the wavefield observed at receivers distributed in the medium:

$$D(\mathbf{r},t) = W(\mathbf{x}=\mathbf{r},t) \ . \tag{2}$$

- In migration, we use the observed data $D(\mathbf{r},t)$ to generate the wavefield $W(\mathbf{x},t)$ at all positions and all times by wave propagation backward in time. The image represents a subset of the wavefield at time zero:

$$R(\mathbf{x}) = W(\mathbf{x} =, t = 0) \ . \tag{3}$$

In both cases, we solve Equation 1 with different initial conditions, but with the same model, $v(\mathbf{x})$ and $\rho(\mathbf{x})$ and with the same boundary conditions.

EXAMPLE

I illustrate the zero-offset modelind and migration methodology using the Sigsbee 2A synthetic model. This model is based on the Sigsbee structure in the Gulf of Mexico and the velocity is illustrated in Figure 2. The model is characterized by a massive salt body close to the water bottom and surrounded by sediments. The salt velocity is 4.5 km/s and the surrounding sediment velocities range from approximately 1.5 to 3.25 km/s.

In this experiment, I consider sources distributed uniformly in the subsalt region of the model. The data are acquired in a borehole array, located at $x = 8.5$ km and in a horizontal array located at $z = 1.5$ km. In order to avoid multiple scattering in the subsurface, I simulate waves with a smooth version of the Sigsbee model, illustrated in Figure 3, and with constant density.

Using the MADAGASCAR program `sfawefd2d`, we can simulate wavefields from the distributed sources. Figures 4a-4h show wavefield snapshots in order of increasing times. We can observe waves propagating from all subsalt sources, interacting with the variable velocity medium and arriving at the vertical and horizontal arrays.

Figures 5a and 5b show the data observed at the horizontal array in variable density and wiggle plotting formats, respectively. Similarly, Figures 6a and 6b show the data observed in the vertical array using the same plotting formats. The data are just subsets of the same wavefields at the respective receiver positions and capture the complications observed in the wavefield, i.e. triplications due to lateral velocity variation.

In zero-offset migration, we backprogate the acoustic wavefields using the acquired data as boundary conditions. The image is the wavefield at time zero. Since we can acquire data at different locations in space, the reconstructed wavefields depend on the acquisition geometry, thus limiting the illumination in the subsurface. Therefore, the migrated images depend on the acquisition array, as illustrated in Figures 7a and 7b for the horizontal and vertical arrays, respectively. We can also obtain images by migrating the data observed in both the horizontal and vertical arrays, as illustrated in Figure 8, thus increasing the acquisition aperture and the subsurface illumination.

CONCLUSIONS

The combination of LaTeX and MADAGASCAR allows geoscientists to generate reproducible documents where the numeric examples can be verified by any user with the same computer setup. This allows for transparent peer-review, for recursive development and for technology transfer between collaborative research groups.

ACKNOWLEDGMENTS

The reproducible numeric examples in this paper use the MADAGASCAR open-source software package freely available from http://www.ahay.org.

REFERENCES

Clærbout, J. F., 1985, Imaging the Earth's Interior: Blackwell Scientific Publications.

LIST OF FIGURES

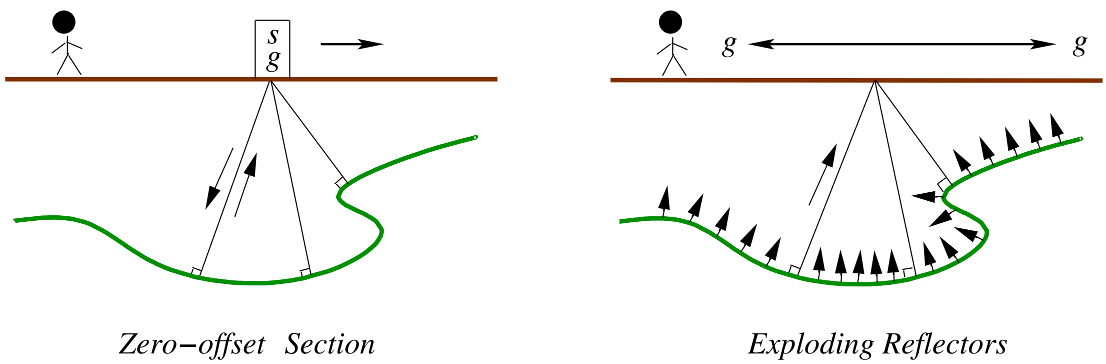

Figure 1: The exploding reflector model (after Clærbout (1985)).
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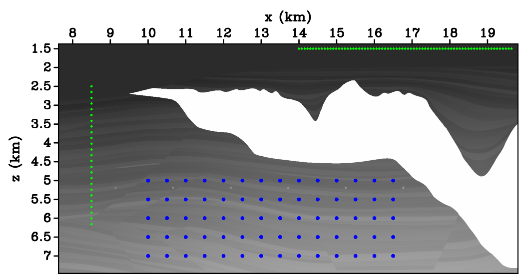


Figure 2: Stratigraphic Sigsbee 2A velocity model.**Sava – Published online**

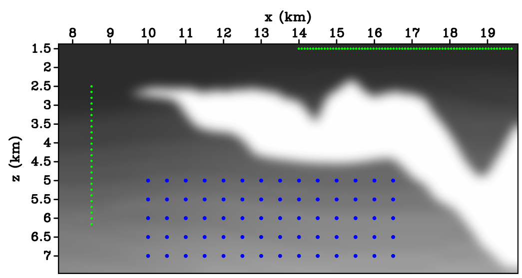


Figure 3: Smooth Sigsbee 2A velocity model.**Sava – Published online**

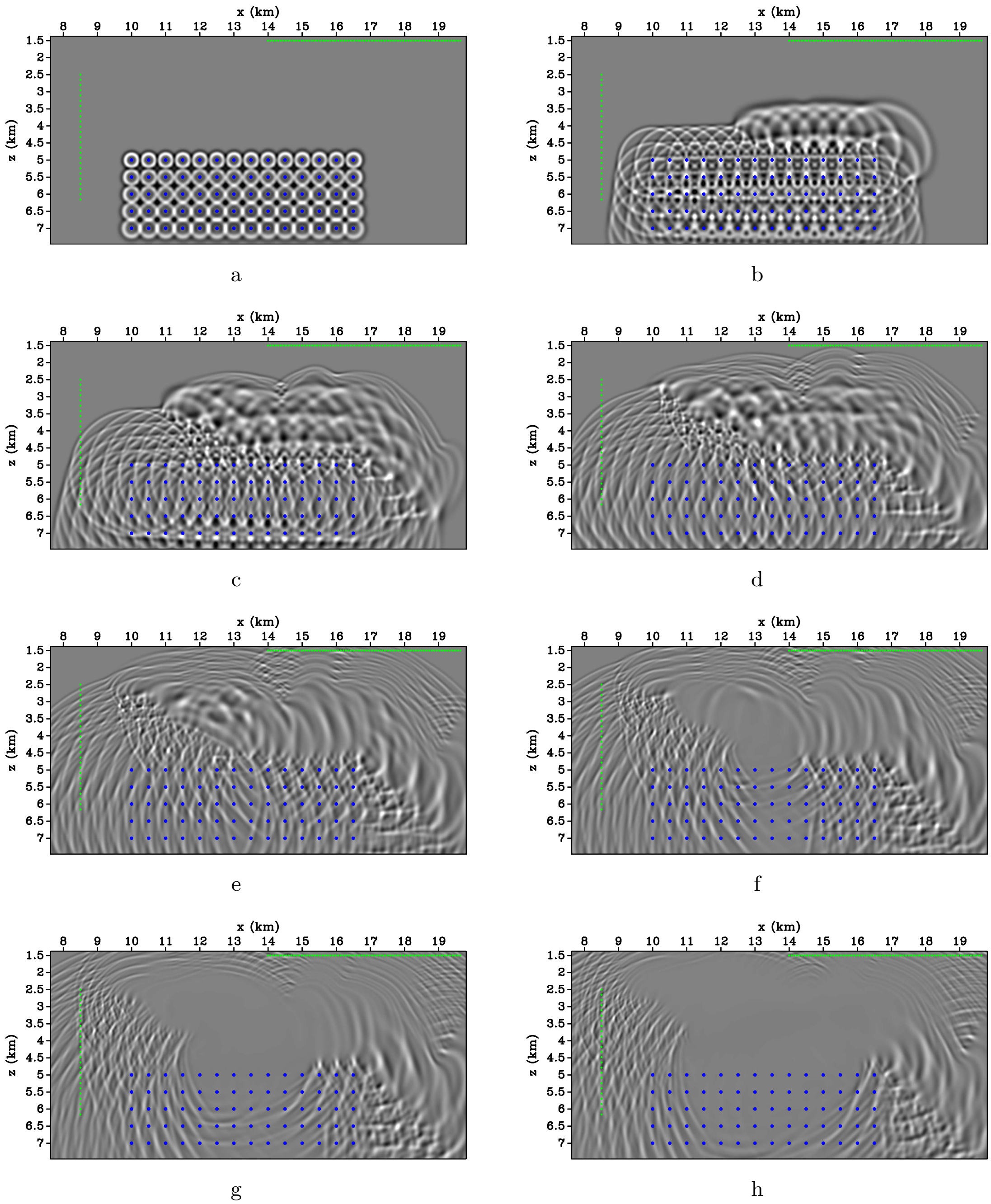
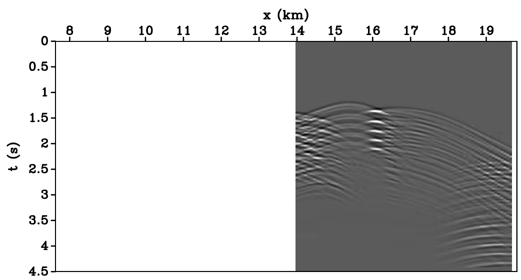
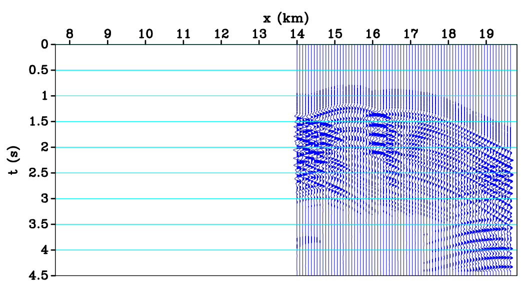


Figure 4: Wavefield snapshots at increasing times.**Sava – Published online**

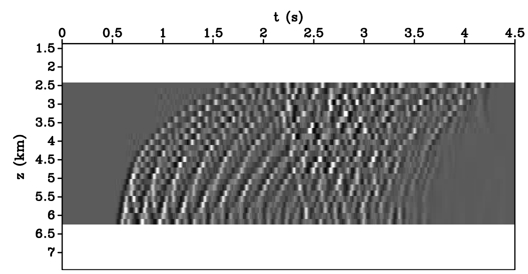


a



b

Figure 5: Acoustic data observed in the horizontal array.**Sava – Published online**



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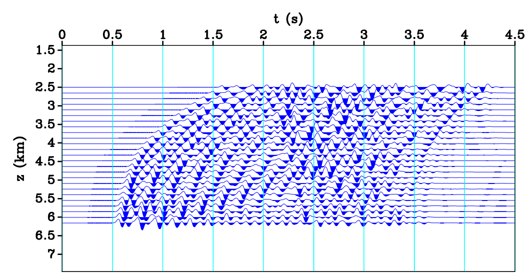

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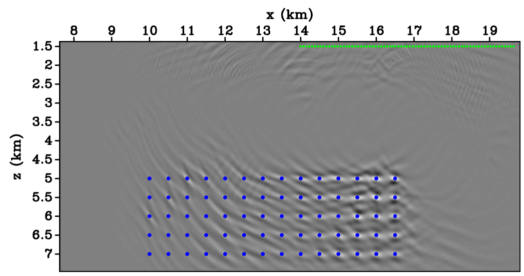
Figure 6: Acoustic data observed in the vertical array.**Sava – Published online**



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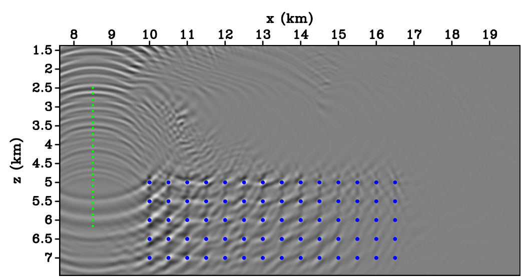

b

Figure 7: Migrated images for data acquired in (a) the horizontal array and (b) the vertical array.
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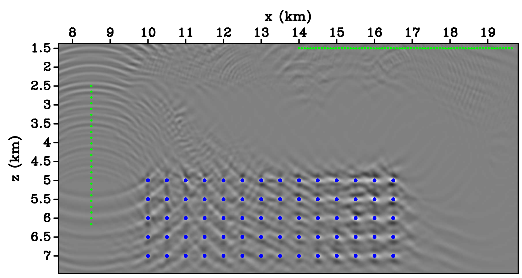

Figure 8: Migrated image for data acquired in both the horizontal and vertical arrays.
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